Achieving Near-Optimum Asymptotic Efficiency and Fading Resistance over the Time-Varying Rayleigh-Faded CDMA Channel

Subramanian Vasudevan and Mahesh K. Varanasi, *Senior Member, IEEE*

***Abstract*— We study multiuser receiver design and analysis for synchronous code-division multiple-access (CDMA) channels with time-varying Rayleigh fading. Starting from an error probability criterion, we first derive a near-optimum receiver for this channel that admits a detector-estimator decomposition, has certain asymptotic optimality properties and a complexity which is independent of the length of the observation interval. The performance of this detector is analytically characterized and contrasted with that of the optimal multiuser detector for the time-invariant (or static) CDMA Rayleigh-fading channel when it is implemented over the time-varying channel. Notable among our conclusions is the fact that, unlike the static channel multiuser detector, the time-varying channel detector is able to withstand not only the estimated interference from the other system users, but also, the residual interference (that cannot be estimated) arising out of imperfect estimation of the interferer fading parameters. Using estimation error covariance information, this detector shows flexibility in accommodating a wide range of interferer fading conditions.**

***Index Terms*—Code-division multiple access, multiuser detection, fading channels.**

I. Introduction

IN this paper, we consider multiuser receivers for symbol-synchronous code-division multiple-access (CDMA) communication over the single path, time-varying, Rayleigh-faded channel with fading memory. In a previous paper [1], we analyzed channels where the fading was Rician and memoryless, i.e., the fading parameters affecting the transmissions of any user over successive bit intervals were assumed independent. As pointed out there, the Rician-fading model is applicable however only to channels, such as those encountered in satellite to land communications, where a steady, direct or specular signal link exists between the transmitter and the receiver. In communication between mobiles and base stations, such a link is unlikely to exist in urban areas. This paper thus constitutes an extension to a realistic fading model for such channels. We work in the context of a synchronous and single-path channel in order to be able to focus on the new issues that arise as a result of the increased complexity of this fading model. As in the Rician case, we expect our analysis here to facilitate the study of the corresponding channels with multipath and/or asynchronism [2]. Additionally synchronous detection in the context of Rayleigh-fading channels is of some interest by itself in light of the fact that the downlink of at least one CDMA system proposed for personal communication services (PCS) is synchronous [3].

Under the constraint of synchronicity, the model of this paper is the most realistic considered to date in the context of Rayleigh-fading channels. Recent work [4] has focused on multiuser detection for an idealized static Rayleigh-fading CDMA channel model. This static or time-invariant channel model assumes that the channel states do not vary over the observation interval and hence that perfect estimates for these states are available at the receiver. In contrast, the time-varying channel model that we consider in this work makes allowance for the dynamic evolution of the channel states. One consequence of operating over such a channel is that these states can, in general, only be partially estimated.

Our work here may be broadly partitioned into two areas: multiuser receiver design for the time-varying Rayleigh-fading CDMA channel and a performance characterization of the receiver designed which includes in particular an assessment in this regard vis-a-vis the optimum multiuser detector proposed for the corresponding static channel when the latter is implemented over the time-varying fading channel under consideration. The approach adopted in this paper to receiver design is to derive receivers from error probability criteria which are formulated recognizing the time-varying nature of this channel. In this regard we follow prior work in the context of the single-user time-varying Rayleigh-fading channel [5], [12] extending in particular the approach in [5] to the multiuser channel. The performance characterization involves establishing the effect of multiuser interference on each detector and evaluating the competitiveness or lack thereof with optimal strategies over single-user channels.

The rest of this paper is organized as follows. After presenting the signal model in Section II, the near-optimum receiver for the time-varying Rayleigh-fading CDMA channel is proposed in Section III. In Section IV-A, we introduce asymptotic performance measures to enable evaluation of

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S. Vasudevan was with the Department of Electrical and Computer Engineering, University of Colorado, Boulder, CO 80309 USA. He is now with US WEST Advanced Technologies, Boulder, CO 80303 USA.

M. K. Varanasi is with the Department of Electrical and Computer Engineering at the University of Colorado, Boulder, CO 80309 USA.

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various candidate detectors for this channel. The application of these measures requires determining binary hypothesis test (BHT) error probabilities for the detectors under consideration. These are detailed, in the context of a general bit-error rate (BER) evaluation procedure for Rayleigh-fading channels, in Section IV-B. Based on these results, we use the measures of Section IV-A to investigate the effect of multiuser interference on the performance of the proposed time-varying channel detector as well as the optimum multiuser detector for the static channel when it is employed over the time-varying channel. Section V presents numerical results that demonstrate the interesting characteristics of the near-optimum receiver designed for the time-varying channel, under realistic channel scenarios while the last section summarizes the results of the paper and introduces possible extensions of this work.

II. THE SIGNAL MODEL

We introduce first the signal model for the time-varying CDMA Rayleigh-fading channel. The received baseband signal is given by

$$r(t) = \sum_{l=0}^{i} \sum_{k=1}^{v} b_k(l)\gamma_k(l)a_k(t - lT)e^{\jmath\phi_k(l)} + n(t) \qquad (1)$$

and is seen to be a superposition of attenuated and phase-shifted versions of the v users' transmissions over a total of $i+1$ bit intervals observed in additive noise $n(t)$ of variance σ^2. The signature signals $\{a_k(t)\}$, time-limited to $[0, T)$, are linearly independent. The fading parameters associated with each user $\{\gamma_k(l)e^{\jmath\phi_k(l)}\}$, assumed constant over the duration of a bit interval, are realizations of zero-mean Gaussian wide-sense stationary (WSS) processes. Further, the fading parameters associated with each user are assumed to be independent of those that affect the other users. A binary phase shift keying (BPSK) modulation format is considered with the kth user's data bit over the lth bit interval $b_k(l) \in \{-1, 1\}$. This choice of a modulation format that requires coherent detection is motivated by the attendant advantages of such detection and made possible by the presence of fading memory over this channel.

We now seek to detect the v users' data transmissions using an optimality criterion based on the observation of $r(t)$ and knowledge of the underlying statistical characterization of the fading processes. This approach follows from our recognition of the fact that it would be unrealistic to assume *a priori* knowledge of the time-varying fading amplitudes and phases necessary for coherent detection and that an approach which in effect uses $r(t)$ and the aforementioned statistics to generate the required estimates of these parameters is more appropriate. It is not unreasonable to expect the availability (or at least the on-line computability) of these statistics for large classes of mobile channels such as those characterized by exponential covariance sequences.

We start by first outlining some notation that organizes the various channel parameters into vectors and matrices. The user data bits, fading parameters, and matched filter outputs over the lth bit interval are organized into the v length column vectors $\boldsymbol{b}(l), \boldsymbol{f}(l)$ and $\boldsymbol{q}(l)$

$$\boldsymbol{b}(l)_k = b_k(l); \quad \boldsymbol{f}(l)_k = \gamma_k(l)e^{\jmath\phi_k(l)};$$
$$\boldsymbol{q}(l)_k = \int_{lT}^{(l+1)T} r^*(t)a_k(t - lT)\,dt. \qquad (2)$$

The v dimensional normalized signal correlation matrix over any bit interval is denoted by R: $R_{kg} = \int_0^T a_k(t)a_g^*(t)\,dt$, and the *a priori* fading correlation sequences are organized as the diagonal matrices $\{V(n)\}$: $V(n) = E[\boldsymbol{f}^*(n')\boldsymbol{f}^T(n'+n)]$.

A discrete-time equivalent model for the received signal in (1) is obtained through the characterization of the vectors $\{\boldsymbol{q}(l)\}$ that form a set of sufficient statistics for the detection problem. $\boldsymbol{q}(i)$ may be written as

$$\boldsymbol{q}(i) = RF^*(i)\boldsymbol{b}(i) + \boldsymbol{n}(i) \qquad (3)$$

where $F^*(i)$: $[v \times v]$, $F^*(i)_{kg} = \boldsymbol{f}^*(i)_k\delta_{k-g}$, and $\boldsymbol{n}(i)$: $[v \times 1]$, $\boldsymbol{n}(i)_k = \int_{iT}^{(i+1)T} n^*(t)a_k(t - iT)\,dt$. $\boldsymbol{n}(i)$ is therefore seen to be a Gaussian random vector with mean and covariance $[\boldsymbol{0}, \sigma^2 R]$.

III. THE NEAR-OPTIMUM RECEIVER

We develop now the multiuser receiver proposed for implementation over our fading channel followed by a justification for this choice of reception strategy. Since, as pointed out before, apriori availability of the channel state information is not assumed, this specification consists of two parts: a multiuser estimator that provides channel-state related information, and a multiuser detector that makes use of this information to detect the transmitted data. Both the estimator and detector are specified for the ith observation interval under the assumption that the data over the previous intervals, denoted $\mathcal{B} = \{\boldsymbol{b}(l): l < i\}$ is known. Additionally, the proposition specifying the detector makes a second assumption which is that the estimator outputs are uncorrelated. The motivation behind specifying the receiver under these assumptions is that they represent conditions under which the receiver is optimal. It must be noted that the implementability of the proposed receiver is *not* predicated on these assumptions holding. The only effect of relaxing these assumptions is to render the receiver near-optimal. At the end of this section, we specify how this receiver would actually be realized in practice.

A. The Detector

The detection rule for the time-varying channel (TVC) is stated by the following proposition.

Proposition 1: Under the assumption that the errors in the estimates of the ith bit interval fading are uncorrelated, the detection component of the receiver for the data over this interval, given the past data $\mathcal{B}$

$$\hat{\boldsymbol{b}}(i) = \arg\min_{\boldsymbol{b}(i)} \mathcal{P}(r(t)/\boldsymbol{b}(i), \mathcal{B}) \qquad (4)$$

may be written in closed form as

$$\Phi_{\text{TVC}}: \hat{\boldsymbol{b}}(i) = \arg\max_{\boldsymbol{b}(i)} 2\Re\{\boldsymbol{b}^T(i)\boldsymbol{y}_f(i)\} - \boldsymbol{b}^T(i)H_f(i)\boldsymbol{b}(i) \qquad (5)$$

where

$$\boldsymbol{y}_f(i) = \hat{F}(i)(I + RC^d(i))^{-1}\boldsymbol{q}(i) \tag{6}$$

$$H_f(i) = \hat{F}(i)(R^{-1} + C^d(i))^{-1}\hat{F}^*(i). \tag{7}$$

$\hat{F}(i)$: $[v \times v]$ is a diagonal matrix of ith bit interval fading estimates and $C^d(i)$: $[v \times v]$ the corresponding diagonal error covariance matrix normalized by the background noise variance σ^2.

Proof: To establish the result, we first recognize that the likelihood (4) may be written as

$$\hat{\boldsymbol{b}}(i) = \arg\max_{\boldsymbol{b}(i)} \int \mathcal{P}(\boldsymbol{q}(i)/\boldsymbol{b}(i), \boldsymbol{f}^*(i))\mathcal{P}(\boldsymbol{f}^*(i)/\mathcal{Q}, \mathcal{B})\, d\boldsymbol{f}^*(i) \tag{8}$$

where $\mathcal{Q}$ is defined as the set of past matched filter outputs: $\{\boldsymbol{q}(l): l < i\}$. It turns out that the above rule admits a closed form expression

$$\begin{aligned}\hat{\boldsymbol{b}}(i) = \arg\max_{\boldsymbol{b}(i)} &- \ln |C(i)^{-1} + R(i)| + \frac{1}{\sigma^2}\\ &\cdot (\boldsymbol{q}^{*T}(i)B(i) + \hat{\boldsymbol{f}}^T(i)C(i)^{-1})(C(i)^{-1} + R(i))^{-1}\\ &\cdot (C(i)^{-1}\hat{\boldsymbol{f}}^*(i) + B(i)\boldsymbol{q}(i))\end{aligned} \tag{9}$$

where $R(i) = B(i)RB(i)$ with $B(i)$: $[v \times v]$ given by $B(i)_{kg} = \boldsymbol{b}(i)_k\delta_{k-g}$. The vector $\hat{\boldsymbol{f}}^*(i)$: $[v \times 1]$, defined as $\hat{\boldsymbol{f}}(i)_k = \hat{\boldsymbol{F}}(i)_{kg}\delta_{k-g}$, and the matrix $C(i)$ are the conditional mean estimate of the ith bit-interval fading parameters and the associated normalized estimation-error covariance, respectively

$$\hat{\boldsymbol{f}}^*(i) = E[\boldsymbol{f}^*(i)/\mathcal{Q}, \mathcal{B}] \tag{10}$$

$$C(i) = \frac{1}{\sigma^2}E[(\boldsymbol{f}^*(i) - \hat{\boldsymbol{f}}^*(i))(\boldsymbol{f}^T(i) - \hat{\boldsymbol{f}}^T(i))/\mathcal{Q}, \mathcal{B}]. \tag{11}$$

The result (9) follows on recognizing first that both the distributions under integration in (8) are Gaussian. Under the specified conditioning, $\boldsymbol{q}(i)$ is a Gaussian random vector with mean $RF^*(i)\boldsymbol{b}(i)$ and covariance $\sigma^2 R$. The second distribution in (8) is the *a posteriori* statistical characterization of $\boldsymbol{f}^*(i)$ and we denote its mean and covariance as $\hat{\boldsymbol{f}}^*(i)$ and $S(i)$ $(= \sigma^2 C(i))$, respectively. We may now use the method of completing the square to evaluate the integral.

Under the assumption that $C(i)$ is a diagonal matrix, the rule (9) simplifies to

$$\begin{aligned}\hat{\boldsymbol{b}}(i) = \arg\max_{\boldsymbol{b}(i)} &\boldsymbol{b}^T(i)\hat{F}(i)C^d(i)^{-1}(C^d(i)^{-1} + R)^{-1}\\ &\cdot C^d(i)^{-1}\hat{F}^*(i)\boldsymbol{b}(i) + 2\Re\{\boldsymbol{b}^T(i)\hat{F}(i)C^d(i)^{-1}\\ &\cdot (C^d(i)^{-1} + R)^{-1}\boldsymbol{q}(i)\}.\end{aligned} \tag{12}$$

Subtracting the data independent term $\boldsymbol{b}^T(i)\hat{F}(i)C^d(i)^{-1}\hat{F}^*(i)\boldsymbol{b}(i)$ from the above expression, using Woodbury's Identity [11], and the definitions of $\boldsymbol{y}_f(i)$ and $H_f(i)$, we get (5). □

B. The Channel Estimator

A description of the receiver structure is not complete without an explicit specification of the fading estimator since it is the estimator outputs that enable coherent detection of data over this channel.

Proposition 2: The estimate $\hat{\boldsymbol{f}}(i)$ of the fading-parameter vector over the ith bit interval, produced by the estimator, is given in closed form by

$$\hat{\boldsymbol{f}}^*(i) = \mathcal{V}^{*T}(V + \sigma^2 BA^{-1}B)^{-1}BA^{-1}\boldsymbol{q} \tag{13}$$

while the residual uncertainty about these estimates is specified by

$$C(i) = \frac{1}{\sigma^2}(V(0) - \mathcal{V}^{*T}(V + \sigma^2 BA^{-1}B)^{-1}\mathcal{V}) \tag{14}$$

where $\boldsymbol{q}$: $[iv \times 1] = [\boldsymbol{q}(0)^T \quad \boldsymbol{q}(1)^T \quad \cdots \quad \boldsymbol{q}(i-1)^T]^T$ is the vector of matched filter outputs through the $i - 1$st bit interval, B, A: $[iv \times iv]$ are block diagonal matrices whose lth diagonal blocks are $B(l-1)$ and R, respectively, and further

$$\mathcal{V} = \begin{bmatrix} V(i) \\ V(i-1) \\ \vdots \\ V(1) \end{bmatrix} \quad \text{and}$$

$$V = \begin{bmatrix} V(0) & V(1) & \cdots & V(i-1) \\ V(-1) & V(0) & \cdots & V(1-2) \\ \vdots & & & \vdots \\ V(1-i) & V(2-i) & \cdots & V(0) \end{bmatrix}. \tag{15}$$

Proof: The joint (normal) distribution of $\boldsymbol{f}^*(i)$ and $\boldsymbol{q}$ (given B) is specified by

$$E\begin{bmatrix} \boldsymbol{q} \\ \boldsymbol{f}^*(i) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \text{and}$$

$$E\begin{bmatrix} \boldsymbol{q} \\ \boldsymbol{f}^*(i) \end{bmatrix}[\boldsymbol{q}^{*T} \quad \boldsymbol{f}^T(i)] = \begin{bmatrix} ABVBA + \sigma^2 A & AB\mathcal{V} \\ \mathcal{V}^{*T}BA & V(0) \end{bmatrix}. \tag{16}$$

The result follows by application of the Gauss-Markov Theorem [9] which specifies the form of, and error covariance associated with, the desired estimator. □

C. Estimator Implementation

The estimator complexity in Proposition 2 is, in general, a polynomial function of the observation interval length. One mechanism for implementation of this estimator, when the fading processes are Markov, is a Kalman filter the complexity of which is independent of this observation interval length [8].

We present below the well-known recursions [10] specifying the estimator for the special case of exponential covariance sequences, i.e., $\{V(n)_{kk} = V(0)_{kk}\xi_k^{|n|}\}$, where the ith interval fading estimates and error covariances are updates of the corresponding quantities for the $i - 1$st bit interval

$$\begin{aligned}\hat{\boldsymbol{f}}^*(i) = &V^N(1)\hat{\boldsymbol{f}}^*(i-1) + V^N(1)S(i-1)\\ &\cdot (S(i-1) + \sigma^2 R(i-1)^{-1})^{-1}\\ &\cdot (B(i-1)R^{-1}\boldsymbol{q}(i-1) - \hat{\boldsymbol{f}}^*(i-1))\end{aligned} \tag{17}$$

and

$$S(i) = V(0) - V^N(1)(V(0) - \sigma^2(\sigma^2 S(i-1)^{-1} + R(i-1))^{-1})V^N(1) \tag{18}$$

where $V^N(1)_{jk} = V(1)_{jk}/V(0)_{jj}$ and the initialization for the recursion is $\hat{\boldsymbol{f}}^*(0) = 0, S(0) = V(0)$. The result may be derived by making use of the matrix inversion lemma [11] to simplify the original specification of the estimator. Note that the estimator complexity is now *independent* of the observation interval and only a polynomial function of the number of system users.

D. Practical Realization of the Receiver

In general, we cannot be assured that the normalized estimator covariance $C(i)$ is diagonal and, therefore, the detection rule which we shall consider in the sequel will be (5) with $C^d(i)_{gh} = C(i)_{gh}\delta_{g-h}$. Thus, while the detector was derived under the assumption of uncorrelated fading estimates, no such assumption is made in actually implementing it over the fading channel. The following lemma provides support however for the use of this detector under this more general scenario with the proviso that the observations over the previous bit intervals be made in light additive noise.

Lemma 1: The detection component of the optimal receiver for data over the ith bit interval given the fading parameters over the previous bit intervals

$$\hat{\boldsymbol{b}}(i) = \arg\max_{\boldsymbol{b}(i)} \mathcal{P}(r(t)/\boldsymbol{b}(i), \mathcal{F}) \tag{19}$$

where $\mathcal{F} = \{\boldsymbol{f}(l): l < i\}$, has the same closed form expression as Φ_{TVC} with the estimate of the fading parameters and their aposteriori (now necessarily diagonal) normalized covariance redefined as

$$\hat{\boldsymbol{f}}^*(i)\ (= \hat{\boldsymbol{f}}^{p*}(i)) = E(\boldsymbol{f}^*(i)/\mathcal{F}) \tag{20}$$

$$C^d(i)\ (= C^p(i)) = \frac{1}{\sigma^2} E((\boldsymbol{f}^*(i) - \hat{\boldsymbol{f}}^{p*}(i)) \cdot (\boldsymbol{f}^T(i) - \hat{\boldsymbol{f}}^{pT}(i)/\mathcal{F}). \tag{21}$$

It turns out that the estimates of the ith bit interval fading, as well as the residual error covariances, based on noisy observations of the fading used in Φ_{TVC} coincide with the estimates and covariances of the previous lemma as the additive noise over the first $i - 1$ bit intervals vanishes. This asymptotic coincidence of these two receivers, or equivalently, the asymptotic optimality of the detector that uses diagonalized error covariances, strengthens our choice of it as a candidate multiuser detector for the time-varying Rayleigh-fading channel.

A further compromise that needs to be made in implementing the receiver is that the past data used to compute the fading estimates and covariance is replaced by an estimate thereof denoted by $\hat{\mathcal{B}}$. Further the entire receiver is implemented recursively.

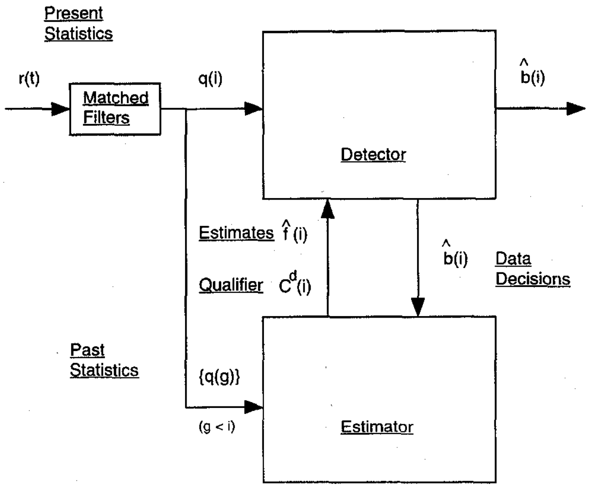

Fig. 1. Detector-estimator decomposition of the proposed decision-directed receiver for the time-varying channel.

Fig. 1 illustrates the near-optimum receiver, as a composition of estimator and detector, and as it would actually be implemented. The *a posteriori* characterization of the ith bit interval fading employed by the detector to detect the ith bit interval data is produced by the estimator component of the receiver based on past matched filter outputs and corresponding data estimates. The complexity of this detector is polynomial in the number of users and *independent* of the length of the observation interval.

IV. RECEIVER PERFORMANCE EVALUATION

An important goal of any performance analysis of a CDMA system is an assessment of the effects of multiuser interference on detector performance. In the context of the time-varying Rayleigh-fading channel, we first recognize that there are two components to this interference; the estimated interference from the other system users and the unmeasured component arising out of imperfect estimates of the interferer fading parameters. Thus the effect of both these components on detector performance needs to be gauged. Previously defined performance measures for Rayleigh-fading channels [4] find application only in the absence of estimation error and hence we are motivated to define afresh, the corresponding measures for our channel.

A. Performance Measures

The regime under which we carry out the required analysis is the high average estimated signal-to-noise ratio (ESNR) region over the ith bit interval, with the additional assumption that the fading over the previous bit intervals is observed noiselessly. Making the latter assumption simplifies our asymptotic analysis of detector performance while still representing a scenario where both estimated as well as unestimated interference are present over the ith bit interval. This is so since the nonflat fading correlation sequences continue to introduce errors in our estimates of the ith bit interval fading even though these estimates are based on perfect knowledge of the past fading. Note further that under this condition, the proposed receiver and Φ_{TVC} coincide with (9).

It must, however, also be recognized that the two sources of performance degradation arising from the detectors' ignoring the correlation between the fading estimates and assuming past decisions to be error-free when in fact they are not, are absent in the regime under consideration and therefore not addressed through the following analytical measures. These issues are dealt with through the numerical results of Section VI.

Under the assumption of noiseless past fading observation, we now introduce the asymptotic efficiency measure [6] for the time-varying synchronous Rayleigh-fading channel with memory, defining it formally, for an arbitrary detector Φ, as

$$\eta_k(\Phi) = \sup\left\{ 0 \le r \le 1; \lim_{\substack{\sigma \to 0 \\ (1/\sigma^2)S^p(i) = \tilde{C}^p(i)}} \frac{P_k(\Phi)}{\frac{1}{2}\left(1 - \frac{1}{\sqrt{1 + \frac{1}{r\lambda_k}}}\right)} \le 1 \right\} \tag{22}$$

where $S^p(i)$ denotes the (now diagonal) estimation error covariance matrix and $\lambda_k = (1/\sigma^2)T^p(i)_{kk}$[1] is the kth user ESNR. The numerator in the above expression is the kth user error probability in the multiuser environment while the denominator is the corresponding BER when transmissions are made in isolation over the static channel with power equal to the user fading-estimate power in the multiuser environment. Further, in taking the limit, the normalized estimation error covariance matrix is held fixed at $\tilde{C}^p(i)$. Since $\eta_k(i)$ is parametrized by the interferer fading-estimate powers $\{T^p(i)_{gg}\}_{g \ne k}$ and the normalized estimator error covariance, this measure can be the basis for assessing the loss in performance due to the estimated interferer signal components as well as the interferer estimation errors.

To isolate the limitations in detector performance due to the estimated interference, we now define a near–far resistance measure for this channel.[2] Formally,

$$\overline{\eta}_k(\Phi) = \min_{\substack{T^p(i)_{gg} \ge 0 \\ g \ne k}} \eta_k(\Phi). \tag{23}$$

At a fixed set of estimation-error covariance to additive-noise variance ratios $\tilde{C}^p(i)$, a detector Φ is said to be near–far resistant, if $\overline{\eta}_k(\Phi) \ne 0$. Detectors for which $\overline{\eta}_k(\Phi) = 0$, for any $\tilde{C}^p(i)$ are denoted as being near–far limited. Such detectors fail to show an inverse rate of BER decay with increasing average ESNR and decreasing estimation error for all the users.

The kth user fading resistance of a detector, defined as

$$\varrho_k(\Phi) = \lim_{\{\tilde{C}^p(i)_{gg} | g \ne k \to \infty\}} \eta_k(\Phi) \tag{24}$$

is our measure for isolating detector performance limitations due to the unestimated interference from the other system users. The fading resistance of a detector is interpreted as the effective rate of BER decay with decreasing additive noise variance in the limit as this noise as well as the estimation error of the user in question vanish, but the estimation of the interferers' fading continues to be in error. We use this measure to isolate the performance limitations imposed by interferer estimation error. At a fixed set of fading-estimate powers $T^p(i)$, a detector Φ is said to be fading resistant if $\varrho_k(\Phi) \ne 0$. Detectors for which $\varrho_k(\Phi) = 0$ whatever the interferer fading-estimate powers, are denoted as being fading limited.

The above set of performance measures is now employed to obtain a complete characterization of candidate detectors over the time-varying fading channel.

B. Binary Hypothesis Test Error Probabilities

Application of the previously defined measures requires the evaluation of error rates associated with binary hypothesis tests for the receivers under consideration. In this section, we derive therefore, some results pertaining to such test probabilities for the receiver Φ_{TVC}. These results are then applied, in the following section, to the performance characterization of this receiver under asymptotic conditions. The approach outlined below is also used to evaluate the performance of the other receiver considered in this paper and we will thus have occasion to refer back to this section.

Lemma 2: In a test between the vectors $\boldsymbol{b}^1(i)$ and $\boldsymbol{b}^2(i)$ using the rule (5), the probability of deciding in favor of $\boldsymbol{b}^1(i)$ when in fact $\boldsymbol{b}^2(i)$ is transmitted is given by

$$P(\pi^{*T}\Omega\pi > 0) = \sum_{g:\varphi_g > 0} \beta_g \tag{25}$$

where on the left-hand side (lhs)

$$\pi = \begin{bmatrix} \hat{\boldsymbol{f}}^*(i) \\ \boldsymbol{f}^*(i) \\ \boldsymbol{n}(i) \end{bmatrix}$$

$$\Omega = \begin{bmatrix} X(1,1) & X(1,2) & X(1,3) \\ X(1,2)^{*T} & 0 & 0 \\ X(1,3)^{*T} & 0 & 0 \end{bmatrix} \tag{26}$$

with

$$\begin{bmatrix} X(1,1) \\ X(1,2) \\ X(1,3) \end{bmatrix} = \begin{bmatrix} B^2(i)G(i)B^2(i) - B^1(i)G(i)B^1(i) \\ (B^1(i) - B^2(i))G(i)B^2(i) \\ (B^1(i) - B^2(i))G(i)R^{-1} \end{bmatrix} \tag{27}$$

and $G(i) = (R^{-1} + C^d(i))^{-1}$. On the right-hand side (rhs)

$$\beta_g = \prod_{h \ne g} \frac{1}{1 - \frac{\varphi_h}{\varphi_g}} \tag{28}$$

and $\{\varphi_h\}$ are the eigenvalues of $\Pi\Omega$ *where* Π is the covariance of π.

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[1] $T^p(i) = E[\hat{\boldsymbol{f}}^{p^*}(i)\hat{\boldsymbol{f}}^{p^T}(i)]$ denotes the (also diagonal) estimated-signal power matrix in the same context.

[2] We give near–far resistance a more restrictive meaning here as opposed to the Gaussian channel to indicate robustness only to the estimated component of the interference.

Proof: That the Hermitian form on the lhs of (25) represents the test statistic follows on substituting for $\boldsymbol{q}(i)$ in the decision statistic using the rhs of (3) and then rearranging the terms in the latter.

Conditioned on the data over the previous bit intervals, the vector π is seen to be a vector of Gaussian random variables with covariance matrix

$$\Pi = \begin{bmatrix} T(i) & T(i) & 0 \\ T(i) & V(0) & 0 \\ 0 & 0 & \sigma^2 R \end{bmatrix} \tag{29}$$

where $T(i)$ is the ith bit interval fading estimate power, and $E[\hat{\boldsymbol{f}}^*(i)\boldsymbol{f}^T(i)/\mathcal{B}] = T(i)$ because $\hat{\boldsymbol{f}}(i)$ is the minimum mean-square error estimator of $\boldsymbol{f}(i)$. The decision variable $\pi^{*T}\Omega\pi$ is thus a Hermitian form in Gaussian variates. An approach for obtaining a closed form expression for the associated error probability in terms of the eigenvalues of the matrix $\Pi\Omega$ is detailed in [7]. The expression on the rhs of (25) is the result stated for the case of distinct eigenvalues. □

Our use of this technique for error probability evaluation and analysis is motivated by the fact that it may be applied to all the detectors we consider and additionally shows some promise in terms of applicability to even more complex channels.

The BER determining eigenvalues $\{\varphi_h\}$ are more conveniently specified as solutions to certain simplified determinantal equations that may be derived from the equation $|\varphi I - \Pi\Omega| = 0$ by using, in addition to elementary determinantal identities, the fact that $V(0) = S(i) + T(i)$ from the orthogonality principle. The following lemma states these equations for the case where the fading over the previous bit intervals is noiselessly observed $(C(i) = C^d(i) = C^p(i), T(i) = T^p(i))$.

Lemma 3: The eigenvalues $\{\varphi_g\}$, that determine the binary hypothesis test error probability (25) for the receiver Φ_{TVC} where past fading is observed noiselessly, are solutions to

$$0 = |\varphi^2 I + 4\varphi T^p(i)G^{\varepsilon p}(i) - 4\sigma^2 T^p(i)G^{\varepsilon p}(i)| \tag{30}$$

where $G^{\varepsilon p}(i) = \varepsilon(i)G^p(i)\varepsilon(i), G^p(i) = (R^{-1} + C^p(i))^{-1}$ and $\varepsilon(i) = (B^2(i) - B^1(i))/2$.

C. The Time-Varying Channel Detector

We start by deriving an expression for the detector Φ_{TVC}'s asymptotic efficiency. We are then in a position to determine this detector's near–far and fading resistance and hence characterize the effect of multiuser interference on it. The development here draws on the results pertaining to the error probabilities of binary hypothesis tests involving this detector, presented in the previous section.

Proposition 3: The kth user asymptotic efficiency of the detector (5) is given by

$$\eta_k(\Phi_{\mathrm{TVC}}) = (R^{-1} + \tilde{C}^p(i))^{-1}_{kk}. \tag{31}$$

Proof: Since the BER of this detector cannot be evaluated in closed form, we derive upper and lower bounds on this BER to yield correspondingly, lower and upper bounds on the detector asymptotic efficiency. The coincidence of these bounds, which follows from the asymptotic tightness of the BER bounds under the specified limit in (22), leads to (31).

The upper bound on the detector BER that we consider is the union bound

$$\mathcal{P}_k^u(\Phi_{\mathrm{TVC}}) = \sum_{\varepsilon \in \mathcal{G}_k(i)} 2^{-w(\varepsilon)} \left(\sum_g \beta_g(\varepsilon) \right) \geq \mathcal{P}_k(\Phi_{\mathrm{TVC}}) \tag{32}$$

where $\mathcal{G}_k(i)$ is the set of error sequences such that the $\varepsilon(i)_{kk} = \pm 1$, the parametrization of the residues $\{\beta_g\}$ by $w(\varepsilon)$ has been made explicit, and $w(\varepsilon)$ is the weight of ε. It turns out that this bound suffices to ensure coincidence of the resulting lower bound on the asymptotic efficiency with the upper bound to be derived later.

We now consider the evaluation of the terms in the upper bound. The equation whose roots are needed is (Lemma 3)

$$\begin{aligned} 0 &= |\varphi^2 I + 4\varphi T^p(i)G^{\varepsilon p}(i) - 4\sigma^2 T^p(i)G^{\varepsilon p}(i)| \\ &= |\varphi^2 I + 4\varphi T^p(i)^{1/2}G^{\varepsilon p}(i)T^p(i)^{1/2} \\ &\quad - 4\sigma^2 T^p(i)^{1/2}G^{\varepsilon p}(i)T^p(i)^{1/2}|. \end{aligned} \tag{33}$$

For the unity weight error sequence $\{\varepsilon(i)_{gg} = \delta_{g-k}\}$, we have all the roots of the previous equation equal to zero, except two, which are the solutions to

$$\varphi^2 + 4\varphi T^p(i)_{kk}G^p(i)_{kk} - 4\sigma^2 T^p(i)_{kk}G^p(i)_{kk} = 0. \tag{34}$$

The corresponding BER is given by

$$P_k^l(\Phi_{\mathrm{TVC}}) = \frac{1}{2}\left(1 - \frac{1}{\sqrt{1 + \dfrac{1}{\lambda_k G^p(i)_{kk}}}}\right). \tag{35}$$

We now obtain the BER expressions corresponding to the error sequences of greater weight. Let ε^a denote a particular error sequence of weight w in $\mathcal{G}_k(i)$. The corresponding $2w$ solutions to (33) are derived from the w equations

$$\varphi^2 + 4\varphi\zeta_h(\varepsilon^a) - 4\sigma^2\zeta_h(\varepsilon^a) = 0 \tag{36}$$

where $\{\zeta_h(\varepsilon^a)\}|_{h=1}^{w}$ are the w nonzero eigenvalues of $T^p(i)^{1/2}G^{\varepsilon^a p}(i)T^p(i)^{1/2}$.[3] The solutions to these equations, for each h, are given by $\varphi = -2(\zeta_h \pm \sqrt{\zeta_h^2 + \sigma_h^2})$ where we have suppressed the argument of ζ_h. The resulting w residues associated with the positive eigenvalues (assuming them distinct) are

$$\begin{aligned} \beta_h = \frac{1}{2}\left(1 - \frac{1}{\sqrt{1 + \dfrac{\sigma^2}{\zeta_h}}}\right) \prod_{\substack{l \neq h \\ 1 \leq l \leq w}} &\left(\frac{1}{1 + \dfrac{\zeta_l + \sqrt{\zeta_l^2 + \sigma^2\zeta_l}}{-\zeta_h + \sqrt{\zeta_h^2 + \sigma^2\zeta_h}}} \right) \\ &\cdot \left(\frac{1}{1 - \dfrac{-\zeta_l + \sqrt{\zeta_l^2 + \sigma^2\zeta_l}}{-\zeta_h + \sqrt{\zeta_h^2 + \sigma^2\zeta_h}}} \right). \end{aligned} \tag{37}$$

Given the closed form expressions (35) and (37) for the terms in (32), we now consider the evaluation of the limit in (22)

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[3] $T^p(i) > 0$ is assumed.

with $\mathcal{P}_k(\Phi_{\mathrm{TVC}})$ replaced by $\mathcal{P}_k^u(\Phi_{\mathrm{TVC}})$ assuming r to be nonzero. Using L'hôpital's rule, we find that the ratio of (35) to the single-user BER approaches $r/(R^{-1}+\tilde{C}^p(i))_{kk}^{-1}$ and further that the corresponding ratios for error sequences of greater weight, involving the sum of terms of the type in (37), approach zero. This yields a lower bound on the user asymptotic efficiency of $(R^{-1}+\tilde{C}^p(i))_{kk}^{-1}$.

The lower bound on the BER is the performance obtained with this receiver when a genie reduces the test to one between the transmitted sequence and one that differs from it in the kth bit only. The bound is given by

$$\mathcal{P}_k^l(\Phi_{\mathrm{TVC}}) = \frac{1}{2}\left(1-\frac{1}{\sqrt{1+\dfrac{1}{\lambda_k(R^{-1}+C^p(i))_{kk}^{-1}}}}\right) \le \mathcal{P}_k(\Phi_{\mathrm{TVC}}) \tag{38}$$

which is the error probability associated with the unity-weight error sequence.

We now replace $\mathcal{P}_k(\Phi_{\mathrm{TVC}})$ in (22) by the lhs of the previous inequality and evaluate the limit therein to be $r/(R^{-1}+\tilde{C}^p(i))_{kk}^{-1}$. Since this ratio is constrained to be less than or equal to one, we have $(R^{-1}+\tilde{C}^p(i))_{kk}^{-1}$ as an upper bound on the detector asymptotic efficiency. Since this expression coincides with the lower bound obtained earlier, it is equal to the user asymptotic efficiency. □

The following corollary derives from the form of the asymptotic efficiency in the previous proposition.

Corollary 1: The receiver (5) is near–far resistant with

$$\overline{\eta}_k(\Phi_{\mathrm{TVC}}) = \eta_k(\Phi_{\mathrm{TVC}}). \tag{39}$$

Proof: Since the detector asymptotic efficiency (31) is functionally independent of $\{T^p(i)_{gg}\}$, the detector asymptotic efficiency is nonzero for all interfering estimate-powers and also the detector near–far resistance. □

Considering now the resistance of the receiver (5) to errors in interfering-fading estimates, the implication of the following proposition is that the BER of this detector shows an inverse decay with increasing average ESNR however large these errors may be.

Proposition 4: The receiver (5) is fading resistant with a kth user fading resistance given by

$$\varrho_k(\Phi_{\mathrm{TVC}}) = \frac{1}{(R^{-1}+\tilde{C}^p(i))_{kk}} > 0. \tag{40}$$

Proof: The detector asymptotic efficiency (31) may be written as

$$\eta_k(\Phi_{\mathrm{TVC}}) = \frac{\mathcal{C}_f(R^{-1}+\tilde{C}^p(i))_{kk}}{|R^{-1}+\tilde{C}^p(i)|} \tag{41}$$

where $\mathcal{C}_f(X)_{kk}$ denotes the cofactor of the kkth element of the matrix X. Further, both the numerator and denominator in (41) may be written out as

$$\mathcal{C}_f(R^{-1}+\tilde{C}^p(i))_{kk} = \prod_{g\neq k}\tilde{C}^p(i)_{gg}+\delta^a \tag{42}$$

$$|R^{-1}+\tilde{C}^p(i)| = (R^{-1}+\tilde{C}^p(i))_{kk}\prod_{g\neq k}\tilde{C}^p(i)_{gg}+\delta^b \tag{43}$$

where the second term in the rhs of each equation is of lower order (in the estimator covariance ratios $\{\tilde{C}^p(i)_{gg}\}$) than the corresponding first term. Substituting the previous expressions in (41) and taking the limit as $\{\tilde{C}^p(i)_{gg}|_{gk}\to\infty\}$, by applying L'hôpital's rule, yields (40). □

To summarize the findings of this section, the partial-estimation channel detector is robust to both estimated as well as unmeasured interference however large the latter may be. Employing the estimation error covariances along with the available fading estimates, this detector is able, in general, to use the latter only to the extent that the overall detection process is aided.[4] Thus it is able to mitigate the effect of those interferers which are associated with large estimation errors while still benefiting from those interferer fading estimates that are reliable.

D. The Static Channel Multiuser Detector

At this point, we consider the optimal detection strategy for the static Rayleigh-fading channel since it is the detector that would be implemented over the time-varying channel if one were to ignore the fact that the fading is now only partially estimable and hence we intend to evaluate its performance over the time-varying channel.

The static channel detector [4] is implemented over the time-varying channel as

$$\begin{aligned}\Phi_{SC}\colon \hat{\boldsymbol{b}}(i) &= \arg\max_{\boldsymbol{b}(i)} 2\Re\{\boldsymbol{b}^T(i)\boldsymbol{y}(i)\} - \boldsymbol{b}^T(i)H(i)\boldsymbol{b}(i)\\ &= \arg\max_{\boldsymbol{b}(i)} 2\Re\{\boldsymbol{b}^T(i)\hat{F}(i)\boldsymbol{q}(i)\} - \boldsymbol{b}^T(i)\hat{F}(i)\\ &\quad\cdot R\hat{F}^*(i)\boldsymbol{b}(i).\end{aligned} \tag{44}$$

Over the static channel, Φ_{SC} is the optimal detector since $\hat{F}(i)$ is exactly equal to $F(i)$. This condition cannot of course be achieved over the time-varying channel. Consequently, we investigate first the question of whether this detector suffers from some fundamental limitation over the time-varying channel.

Proposition 5: The kth user BER of the detector Φ_{SC} approaches an error floor over the time-varying Rayleigh-fading channel as the additive noise vanishes, even if the estimator over this channel operates in a noiseless environment and the estimate of the desired user's fading is perfect, i.e., $\hat{F}(i)=\hat{F}^p(i), S(i)=S^p(i), T(i)=T^p(i)$, and further $S^p(i)_{kk}=0, T^p(i)_{kk}=V(0)_{kk}$, except when all the interferers are either perfectly estimated or orthogonal to the kth user.

Proof: To establish the result of the proposition we seek a test between two sequences that differ in the kth bit that has an error floor since this would imply a floor on the BER of Φ_{SC}; the error probability of this test being a lower bound on that of the latter.

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[4] This may be demonstrated by evaluating the fading resistance of this detector for the case where some of the interferers are perfectly estimated and showing it to be larger than the rhs of (40).

We can obtain the counterpart of the determinantal equation (30) of Section IV-B, that yields the eigenvalues which in turn determine the detector's probability of error for a BHT between sequences, for this case as

$$\begin{aligned} 0 = |&\varphi^2 I + 4\varphi T^p(i)\varepsilon(i)R\varepsilon(i) - 4\sigma^2 T^p(i)\varepsilon(i)R\varepsilon(i) \\ &- 4T^p(i)\varepsilon(i)RS^p(i)R\varepsilon(i)|. \end{aligned} \tag{45}$$

The equation follows by first particularizing Lemma 2 for this case, of a test using Φ_{SC} implemented over the time-varying channel, by setting $C^d(i)$ identically equal to zero to obtain the corresponding matrix Ω. Next the corresponding product $\Pi\Omega$ is computed and the approach outlined preceding Lemma 3 employed to obtain the required determinantal equation.

Considering the unity-weight error sequence $\{\varepsilon(i)_{gg} = \delta_{k-g}\}$, we find that all the roots of the previous determinantal equation are zero except for two, and the result is a test BER of

$$\mathcal{P}_k^l(\Phi_{\mathrm{SC}}) = \frac{1}{2}\left(1 - \frac{1}{\sqrt{1 + \dfrac{1 + \boldsymbol{r}(k)^{*T} C^p(i)\boldsymbol{r}(k)}{\lambda_k}}}\right) \tag{46}$$

where $\boldsymbol{r}(k)$ is the kth column of R. It is clear that the above probability does not go to zero even as the background additive noise vanishes. Φ_{SC} thus suffers from an error floor except under the conditions mentioned in the statement of the proposition. □

While the previous proposition establishes a fundamental problem associated with the use of the detector Φ_{SC} over the time-varying channel and the results of the previous section demonstrated the viability of the detector Φ_{TVC}, we would still like to formally relate the asymptotic performances of these two detectors. The following development, to this end, makes use of the unity-weight error sequence test error probability for this detector derived in the proof of the previous Proposition and the asymptotic efficiency expression for the time-varying channel detector stated in Proposition 3.

Proposition 6: The asymptotic efficiency of the detector Φ_{SC} over the time-varying Rayleigh-fading channel is upper bounded by that of the detector Φ_{TVC}

$$\eta_k(\Phi_{\mathrm{SC}}) \le \eta_k(\Phi_{\mathrm{TVC}}). \tag{47}$$

Proof: To establish the inequality, we use the lower bound on the BER of Φ_{SC} given by (46) to obtain the following upper bound on its asymptotic efficiency

$$\eta_k(\Phi_{\mathrm{SC}}) \le \frac{1}{1 + \boldsymbol{r}^{*T}(k)\tilde{C}^p(i)\boldsymbol{r}(k)}. \tag{48}$$

We then write

$$\frac{1}{1 + \boldsymbol{r}^{*T}(k)\tilde{C}^p(i)\boldsymbol{r}(k)} = \frac{1}{\boldsymbol{r}^{*T}(k)(R^{-1} + \tilde{C}^p(i))\boldsymbol{r}(k)}. \tag{49}$$

Now, in order to demonstrate that this quantity is at most equal to $\eta_k(\Phi_{\mathrm{TVC}})$,, i.e.,

$$\frac{1}{\boldsymbol{r}^{*T}(k)(R^{-1} + \tilde{C}^p(i))\boldsymbol{r}(k)} \le (R^{-1} + \tilde{C}^p(i))^{-1}_{kk} \tag{50}$$

we recall that under the constraint $\boldsymbol{r}(k)_k = 1$,

$$\frac{1}{(R^{-1} + \tilde{C}^p(i))^{-1}_{kk}} \le \boldsymbol{r}^{*T}(k)(R^{-1} + \tilde{C}^p(i))\boldsymbol{r}(k) \tag{51}$$

and that at the minimizing point, the lhs of (50) is maximized and equals the rhs. The result follows and we are able to conclude hence that the detector Φ_{SC} is uniformly outperformed by the detector Φ_{TVC} in the high ESNR regime. □

The upper bound on the asymptotic efficiency of Φ_{SC} presented in the previous proof can be used to show that the fading resistance of this detector is zero for all interfering fading-estimate energies (with the same exceptions as in the statement of the Proposition 5) and that therefore this detector is fading limited except under these conditions.

The previous results are significant: they demonstrate that an *ad hoc* concatenation of a detector whose optimality is premised on the availability of perfect fading estimates with an estimator that does not provide such estimates for one or more of the interfering users, does in fact seriously limit the performance of even a user for whom perfect fading estimates are available.

V. NUMERICAL RESULTS

The asymptotic characterizations of detectors obtained in the previous sections bring insight about detector behavior in the high average ESNR region under the assumption of noiseless past fading observation which may be seen as including the assumption of an absence of demodulation errors in the past data. In this section, we investigate the performance of the receivers in the moderate average SNR regime, where in addition, the latter two assumptions are relaxed, with the primary aim of establishing the value of strategies derived for the time-varying channel under realistic channel conditions. In the process, we also illustrate the relevance of our analytical results for evaluating detector performance in realistic scenarios. For the examples to follow, we follow [5] in assuming exponentially decaying fading processes.

The equation-level simulations here are implemented using either FORTRAN and IMSL routines or MATLAB. N independent streams of matched filter outputs with each stream corresponding to an $i+1$ bit-interval observation period, are generated using realizations of data, fading parameter, and additive noise, random variables with appropriate underlying temporal statistics. The receivers are implemented with these matched filter outputs as inputs (a fixed number of initial receptions in each realization serving as a training sequence), and, the "transmitted" data detected. The ith bit interval decision errors are summed and divided by N to yield an estimate of the receiver BER $P_k(i) = p$. The choice of N is made by recognizing that this estimate has a covariance of $p(1-p)/N$ and thus, if a standard deviation of xp is desired for the user BER, the number of trials N must be greater than or equal to $(1-p)/x^2p$.

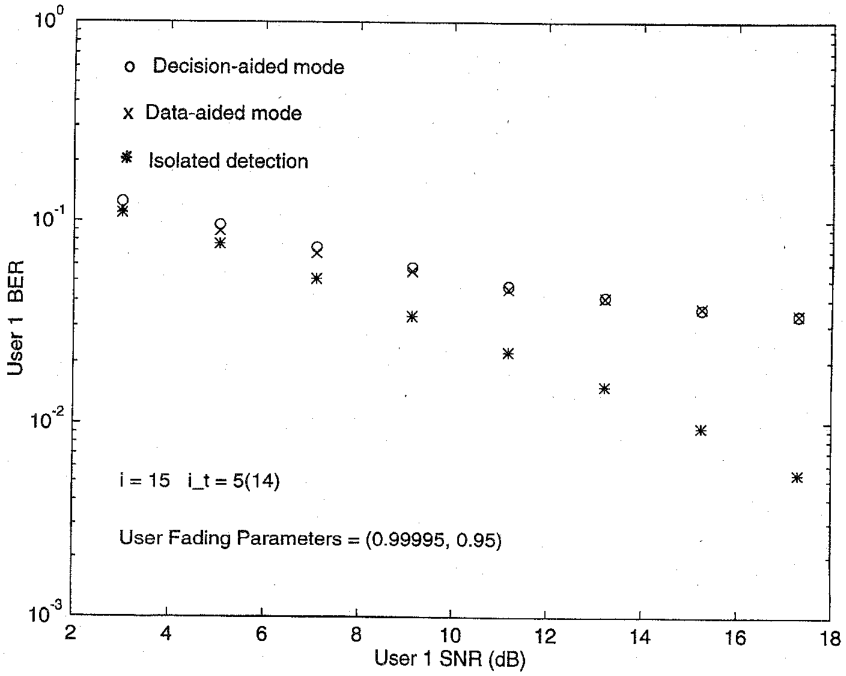

Fig. 2. Impact of multiuser interference on the conventional detector.

In Figs. 2–7, we consider a two-user channel and plot the first user BER[5] (except for Fig. 6) over the ($i = 15$)th bit interval, for each of the receivers considered. The last bit interval is chosen so that the effect of past decision errors is reflected in the performance results. The other channel parameters are denoted as follows: the training sequence length by i_t, the user correlations by $\{\rho_{km}\}$, and the fading covariance sequence by $V(n)_{kk} = V(0)_{kk}\xi_k^{|n|}$. In the simulations to follow, the estimator for each receiver operates on the past matched filter outputs in one of two modes, the first, a data-aided mode where it also has access to the past transmitted data, i.e. the fading estimates are based on $\mathcal{Q}$ and $\mathcal{B}$, and the second, a decision-directed mode where after initial training, the estimator uses the past detector decisions $\hat{\mathcal{B}}$ to update the fading parameter estimates as well as compute the error covariance (where necessary).[6]

In Fig. 2, we plot the BER of the single user detector,[7] implemented with the optimal multiuser estimator, versus average SNR, under each of the modes mentioned above, assuming an interferer half as strong as the desired user: $V(0)_{11} = 1.0, V(0)_{22} = 0.5$, fading covariance parameters: $\xi_1 = 0.99995, \xi_2 = 0.95$, and normalized correlation: $\rho_{12} = 0.5$. The BER of the detector in isolation is plotted as a reference. It is clear that even under the optimistic scenario of no past demodulation error ($i_t = 14$), weak interferer, and availability of optimal fading estimates, the conventional detector BER approaches an unacceptably large error floor. In the decision-directed mode ($i_t = 5$), there is a further worsening of performance.

Fig. 3 is a set of BER versus SNR plots for the static or perfect-estimation channel detector Φ_{SC}, again implemented with the optimal estimator and knowledge of previously transmitted data, for three values of interfering fading covariance parameters $\xi_2 = 0.995$, 0.95, and 0.875, with $V(0)_{11} = V(0)_{22} = 1.0, \rho = 0.5$ and $\xi_1 = 0.9995$. We find that while this detector can cope with a more adverse channel than the conventional detector, the plots clearly bring out the ability of unmeasured interferer fading to limit desired user performance even in a data-aided mode and illustrate the main problem associated with the use of this detector over this channel.

In Figs. 4 and 5, we contrast the performance of the time-varying or partial-estimation channel receiver (whose fading resistance approaches 0.75 here) with that of the perfect-estimation channel receiver (whose fading resistance is zero), implemented as before, both operating in the decision-directed mode for two cases of user fading covariance parameters $(\xi_1, \xi_2) = (0.99995, 0.95)$ and $(0.9995, 0.85)$. and the same values for the other channel parameters. What emerges from these plots is that the time-varying channel detector consistently outperforms the time-invariant channel detector at moderate SNR's. The additional plot of user BER averaged over the entire observation interval in Fig. 5 shows that the BER over the fifteenth bit interval provides a conservative estimate of the improvement that can be expected by use of the proposed partial-estimation channel recevier.

One point to note with regard to the previous plots is that we have focused on the performance of the slow fading user. This is because the fast-fading user does poorly even in a single-user environment and the use of multiuser detectors

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[5] These are obtained through 10 000 run simulations giving us a standard deviation of 3.16×10^{-4} for $p = 10^{-3}$.

[6] The estimator however has access to the statistical characterization of each of the users' fading processes.

[7] This detector is included here largely as a justification for the restriction of the analysis in this paper, to multiuser approaches only for the time-varying Rayleigh-fading channel. An analytical treatment of its limitations over this channel may be found in [14].

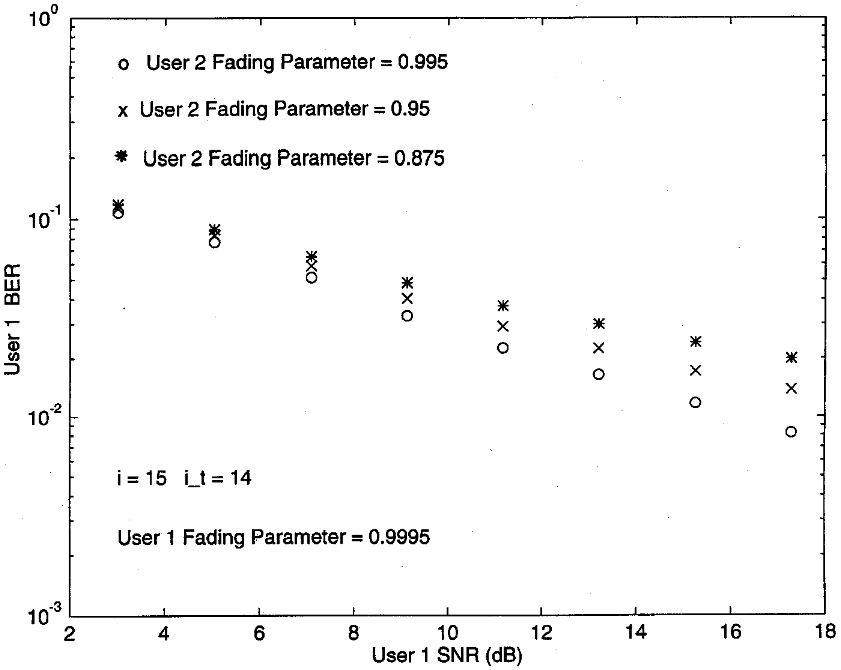

Fig. 3. Effect of interferer estimation error on the perfect-estimation channel detector.

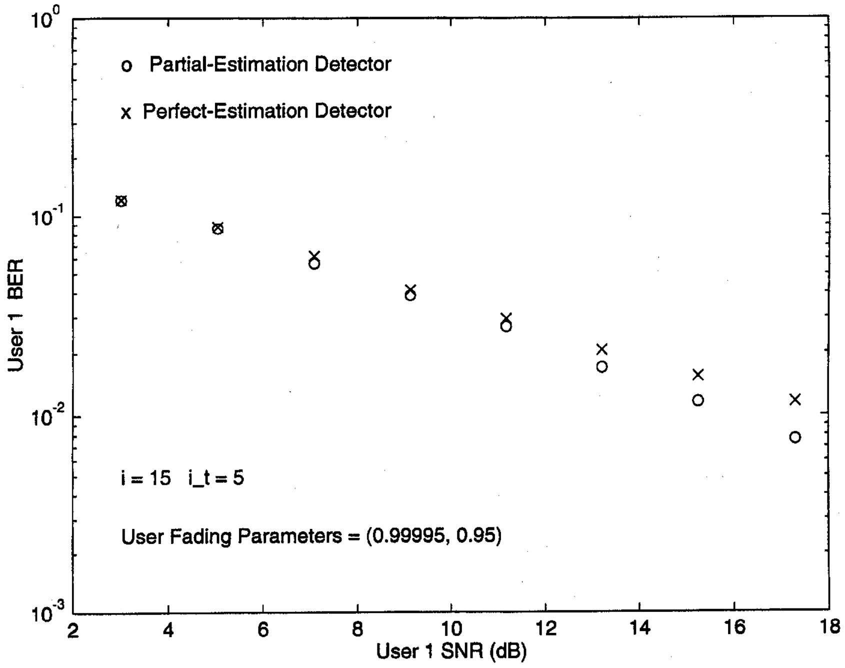

Fig. 4. Comparison of the perfect- and proposed partial-estimation channel detectors.

cannot be expected to improve such a user's performance. This is illustrated in Fig. 6 where it can be seen that the performance of both perfect- and partial-estimation receivers nearly coincide for this user for the fading scenarios of both Figs. 4 and 5.

Also to be noted is the fact that the effect of noisy observation of fading has been observed to be small in such cases, diminishing with increasing SNR.

The plots of Fig. 7 demonstrate that there exist realistic channel conditions under which the time-varying channel detector's performance is not significantly degraded by the diagonalization of the estimator error covariance even at moderate SNR's.

Figs. 8 and 9 deal with the effect that the number of users have, on the performance of the proposed partial-estimation channel receiver. This is first illustrated through the asymp-

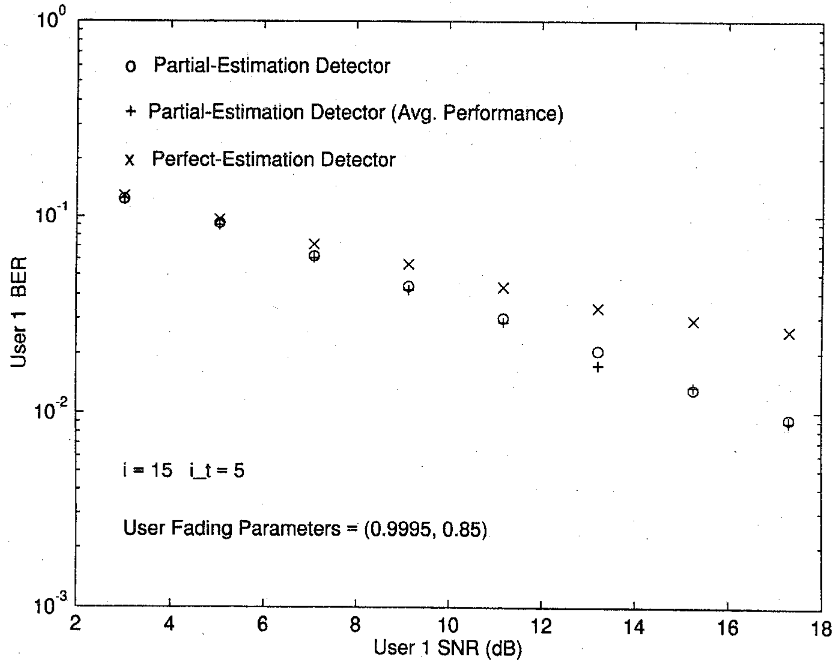

Fig. 5. Comparison of the perfect- and proposed partial-estimation channel detectors.

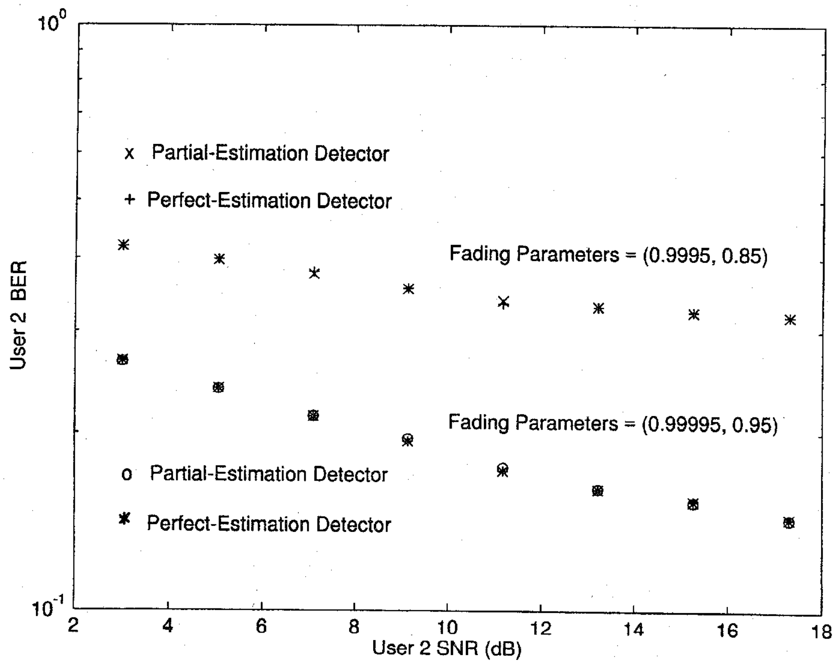

Fig. 6. Comparison of the perfect- and proposed partial-estimation channel detectors for the fast-fading user.

totic performance measures introduced in the paper and then followed up by error probability results. The case of length 31 Gold sequences is considered yielding the correlation matrix for the signals of the users, see (52) at the bottom of the next page.

The asymptotic efficiency trends are shown in Fig. 8 for two cases. The first is specified by $\tilde{C}^p(i)$ = Diag [0.5 5 0.5 0.5 0.5 0.5] and the second by $\tilde{C}^p(i)$ = Diag [0.5 0.5 5 0.5 0.5 0.5]. Apart from the obvious (but relatively small) decrease in the values of the asymptotic performance measures as the number of users is increased, we note that 1) the asymptotic efficiency which is a finer characterization of detector performance reflects the effect of the actual fading levels of the interfering users, and 2) the amount of performance degradation is dependent on how correlated the signal of the added user (user 2 in case 1 and user 3 in case 2) is to that

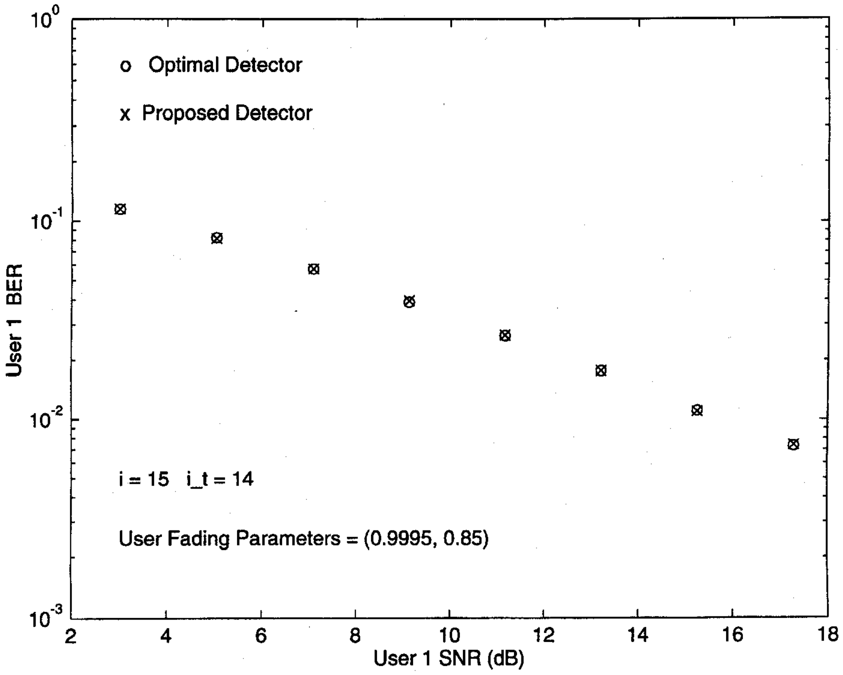

Fig. 7. Impact of error covariance matrix diagonalization on partial-estimation channel detector performance.

of the user under consideration (user 1). The fading resistance reflects the worst case of all the interferers being heavily faded and provides a lower bound on achievable performance.

Fig. 9 shows the impact of adding two users (users 3 and 4), on the BER's of a two-user system (users 1 and 2). Users 2, 3, and 4 have twice the power of user 1. The worsening in performance for this slow-faded user is seen to be marginal and that for the relatively fast-faded user (user 2), negligible. Of course, as we add more users one can expect to see more significant degradation. It bears noting however that the proposed receiver is quite robust in systems employing signals with such moderate correlations.

VI. CONCLUSION AND FUTURE WORK

In this paper, we have formulated and provided solutions to the problem of multiuser reception over Rayleigh-faded channels with time-varying fading, in the synchronous context. Our approach has been one of detecting users transmissions over a given bit interval, without assuming *a priori* knowledge of any part of the channel states, relying instead only on the entire received signal and the underlying statistical characterization of the fading. This is in contrast to simultaneously transmitting a pilot tone to enable coherent communication over such a channel which is power-inefficient.

Two key findings that have resulted from our having taken this approach are the following.

First, the performance-optimizing time-varying channel multiuser receiver has been found to differ from its static counterpart in the fundamental sense that the estimation component supplies, and the detection component uses, not only the channel estimates but also a qualifier about the reliability of this estimate (an error covariance).

Second, we have the result that this receiver is robust, in an asymptotic sense, to both the estimated as well as unmeasured interference components, as opposed to the optimum receiver for the static channel which is assured no immunity to the latter. We have thus established that error floors due to unmeasured other-user interference are not an intrinsic characteristic of multiuser time-varying fading channels in that they can be removed through appropriate receiver design.

Among further issues that merit consideration in light of this work are efficient implementation algorithms for both multiuser estimators and detectors for specific channel models. Also of interest are lower complexity (decorrelative) receivers

$$\int \begin{bmatrix} s_1(t) \\ s_2(t) \\ s_3(t) \\ s_4(t) \\ s_5(t) \\ s_6(t) \end{bmatrix} [s_1(t) \quad s_2(t) \quad s_3(t) \quad s_4(t) \quad s_5(t) \quad s_6(t)] = \begin{bmatrix} 1 & .2258 & -.0323 & -.1613 & .2258 & -.0323 \\ .2258 & 1 & -.2903 & .2258 & -1.613 & .0323 \\ -0.323 & -.2903 & 1 & -.0323 & -.0323 & .0968 \\ -.1613 & .2258 & -0.323 & 1 & .2258 & -0.323 \\ .2258 & -.1613 & -.0323 & .2258 & 1 & -.2903 \\ -.0323 & -.0323 & 0.968 & -.0323 & -.2903 & 1 \end{bmatrix} \tag{52}$$

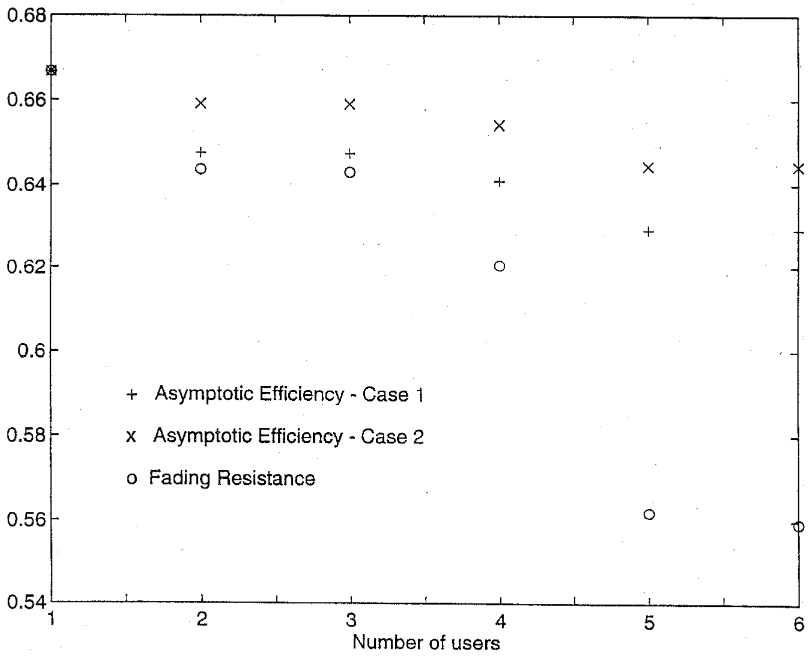

Fig. 8. Variation of asymptotic efficiency and fading resistance of the proposed partial-estimation channel detector with number of users.

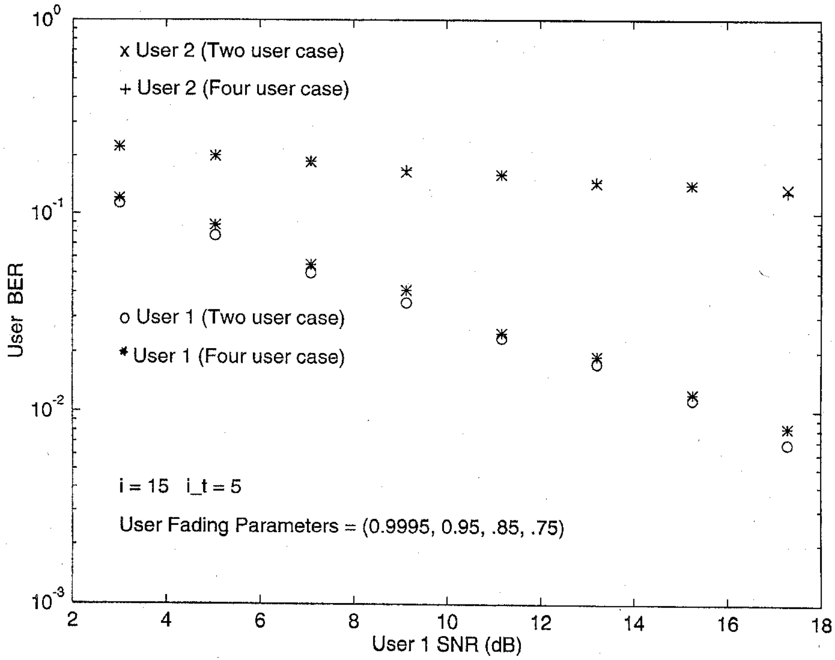

Fig. 9. Impact of additional users on the error probability of the proposed partial-estimation channel detector.

for these channels, such as the detector in [13] or the decorrelating estimator-detector derived and characterized in [15]. These would also be expected to mitigate, if only suboptimally, the multiuser interference problems over these channels.

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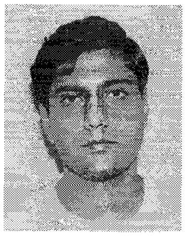

Subramanian Vasudevan received the B. Tech. degree in electrical engineering from the Indian Institute of Technology, Bombay, India, in 1988, and the M.S. and Ph.D. degrees from the University of Colorado, Boulder, CO, in 1990 and December 1993, respectively.

Since 1994, he has been with US WEST Advanced Technologies, Boulder, CO, where his research interests have focused on interference-mitigation techniques for unlicensed spread-spectrum systems, CDMA wireless local loop applications, and most recently, CDMA network optimization.

Mahesh K. Varanasi (S'87–M'89–SM'95), received the B.E. degree in electronics and communication engineering from Osmania University, Hyderabad, India, in 1984, and the M.S. and Ph.D. degrees in electrical engineering from Rice University, Houston, TX, in 1987 and 1989, respectively.

In 1989, he joined the faculty at the University of Colorado, Boulder, where he is currently an Associate Professor of Electrical and Computer Engineering. His teaching and research interests are in the areas of communication theory and signal processing, with a special interest in applications to wireless personal, mobile, and indoor communications. His recent research has been in the areas of coherent and noncoherent multiuser detection, single-and multiuser equalization, receiver designs for fading channel CDMA communications, and signal design and power control for multiuser receivers.