ECEEN 3810 Homework #1

Professor David G. Meyer

September 13, 2010.
Due: September 20, 2010

1. A certain experiment has a sample space \( S \). Suppose \( A, B, C, \) and \( D \) are events and that we know:
   - \( P[A] = 1/8; P[B] = 1/3; P[C] = 1/2, \) and \( A \cup D = C, \)
   - \( A \) and \( B \) are independent, and
   - \( B \) and \( C \) are mutually exclusive.

   (a) If you are told \( A \) and \( D \) are mutually exclusive determine \( P[D] \). If you do not know that \( A \) and \( D \) are mutually exclusive, what’s the most you can say about \( P[D] \)? Explain.
   (b) What is \( P[A \cap B] \)? Why?
   (c) What is \( P[B^c \cap C] \)? Why?
   (d) Does \( B \cup C = S \)? Why or why not?

2. An urn contains 6 red balls, and 10 white. Four balls are drawn from the urn without replacement. The draws are completely random – in other words, all balls left in the urn are equally likely to be grabbed on a given draw.

   In all the problems below, please explain the taxonomy of the counting problem (i.e. ordered/not-ordered, etc.) that you chose as appropriate before doing the count.

   (a) Neglecting order, and recording ONLY the color of the ball drawn, how many unique draws are possible? This count is treating the red balls as indistinguishable (ditto the white) and will not be treatable easily by any formulas or techniques demonstrated so far in class. Do this count by “brute force”. Note you wouldn’t normally do this kind of count if your intentions were to use the count in computing some type of probability. If you want to see a more general, elegant approach than “brute force” to doing this count, then see Section 1.6 of the textbook.
   (b) What is the probability of drawing exactly two red balls?
   (c) What is the probability of drawing at least two red balls?
   (d) What is the probability of drawing a white ball on the third draw?
   (e) What is the probability of drawing exactly two red balls given that a red ball was the first draw?

3. Suppose we take a coin and flip it 5 times, recording the ordered sequence of heads and tails that comes up. The cardinality of the sample space is then clearly \( 2^5 = 32 \). If the coin is ‘fair’ (i.e. equally likely to land heads or tails on any flip), then this experiment generates an equally-likely outcome sample space.\(^1\)

---

\(^1\)A fact we will prove rigorously when we study the theory of independent trials. If the coin is not fair, then this experiment does \textit{not} generate an equally-likely outcome sample space, but we shall show the theory of independent trials will still allow us nicely to compute probabilities for it.
(a) For a fair coin, what is the probability of exactly four heads coming up in the 5 flips?

(b) For a fair coin, what is the probability of exactly four heads coming up in the 5 flips given that the first two flips are heads?

4. An urn contains 12 Type A coins and 6 Type B coins. Type A coins are fair. Type B coins, however, come up heads with an increased probability of 3/4.

(a) A coin is drawn at random from the urn and then flipped. What is the probability the flip comes up heads? Hint: Not an equally-likely outcome sample space problem, so don’t try counting. Instead don’t bother to explicitly list/define a sample space, but think about conditional probabilities.

(b) (Optional exercise). Suppose two coins are drawn without replacement and each is flipped. What is the probability that both flips come up heads?