1. PK syndrome is a rare condition among University Professors that makes them write gruesome images on the chalkboard when they hear their probability class “improvising” on a problem instead of using the tools and techniques that have been put forth in the homeworks and class lectures. There is a test for PK. If a Professor has PK, then 85% of the time the test shows up as positive. However, 15% of the time, the test will show positive anyway even when the tested Professor does not have PK.

What must be the actual prevalence of PK among University Professors in order for the test to be better than merely flipping a coin to decide? Show your work.

2. A Digital communication channel has a the probability of $p_b$ in making a bit error (i.e. with probability $p_b$ a sent “1” is received as a zero by the receiver or vice-versa. The probability of making an error on any given bit is assumed to be independent of what happened when the previous bit was sent.

We need to send 4-bit words over the channel and have two choices:

- We can send the words unadulterated just as 4 bits. In this case, the data is corrupted if any bit is in error, or
- We can use a coding scheme and code the word into a 7 bit codeword and send 7 bits. Because the code has some error correction capability, in this case the data is corrupted only if two bits are in error.

(a) Assume that an error is modeled by exactly one bit being in error. Under this assumption the probability the data is corrupted for the non-coded case is $\binom{4}{1}p_b(1-p_b)^3$ which is obtained quickly using repeated trial theory. Find an expression for the probability that the data will be corrupted if we use the coding scheme assuming corruption occurs when exactly two bits are in error.

(b) From your results in part 2a find for what range of $p_b$ the coding scheme beneficial.

(c) Derive the error probabilities for coded and non-coded transmission under the more accurate assumption that data is corrupted in the non-coded case when at least one bit is corrupted, and in the coded case when at least two bits are corrupted.

(d) (Optional Exercise) Using your error expressions from part 2c, find the range of $p_b$ under which coding is beneficial. Note: You will probably need some computer numerical help to solve for the range of $p_b$. 
3. A discrete random variable $X$ can take on the values $-1, 2, 3, \text{ or } 5$. The CDF (cumulative distribution function), $F_X(x)$, for $X$ is shown in Figure 1 and it takes on the values

(a) What is the probability $P[X \leq 3/2]$?
(b) What is the probability $P[-1/2 < X \leq 7/2]$?
(c) Compute the expected value $\mu = E[X]$.
(d) Compute the variance $\sigma^2 = \text{Var}[X]$

4. A continuous random variable $Y$ can take on any value in the interval $[0, 5]$, but only takes values in that interval. It is known that it’s PDF (probability density function), $f_Y(y)$, has the form

$$Ay(5-y)$$

for some positive constant $A$.

(a) Determine the value of $A$. Use that value in the parts below.
(b) Find the expectation $\mu = E[Y]$
(c) What is the probability $P[Y > 2]$?
(d) What is the probability $P[1 < Y \leq 3]$?
(e) What is the probability $P[Y > 10]$?