Feedback analysis and compensator design example:

Investigation of an op amp lead compensator circuit.

The op amp in the parts kit (ADL 2702) is a CMOS rail-to-rail input op amp with 1 MHz gain-bandwidth product.

\[ \text{Gain} = \frac{1}{1 + \frac{s}{\omega_p}} \]

\[ f_p = \frac{\omega_p}{2\pi} = 2 \text{ Hz} \]

\[ G_o = 5 \times 10^5 \Rightarrow 114 \text{ dB} \]

The gain at 1 MHz is

\[ \left| G_o(s) \right| = \frac{5 \times 10^5}{5 \times 10^5} = 1 \]
Let's consider a lead compensator (PD) having a transfer function
\[ G(s) = A \frac{1 + \frac{s}{\omega_p}}{1 + \frac{s}{\omega_p}} \]

\( |G| \)

\( \angle G \)

Such circuits are sometimes added to feedback loop compensators, to improve phase margin.

A realization using an inverting amplifier:

\[ G(s) = -\frac{Z_2}{Z_1} = \frac{V_o}{V_i} \]

\( \|G\|_{dB} = \|\frac{Z_2}{Z_1}\|_{dB} = \|Z_2\|_{dB} - \|Z_1\|_{dB} \)

(this also adds a minus sign)
Let's make the impedances do this:

\[ \text{mag dB} \]

\[ |Z_2| \text{dB} \]

\[ |Z_2| \text{dB} - |Z_3| \text{dB} = |G| \text{dB} \]

\[ f_p \]

\[ -20 \text{dB/dec} \]

Then \( |G| \text{dB} \) is

Realization of \( Z_1, Z_2 \): a parallel \( R \) and \( C \) will work

\[ Z \rightarrow R \rightarrow C \]

\[ |Z| \text{; parallel combination so select smallest} \]

At \( f_o \): asymptotes intersect

\[ R = \frac{1}{2\pi f_o C} \Rightarrow f_o = \frac{1}{2\pi RC} \]

Suppose we want \( A = 10 \Rightarrow 20 \text{ dB} \)

\[ f_2 = 10 \text{ kHz} \]

\[ f_p = 100 \text{ kHz} \]
Let's choose (arbitrarily) $R_1 = 1 \, \text{k}\Omega$.

Then $Z_1$ and $Z_2$ should be:

1. $Z_1 = 1 \, \text{k}\Omega$
2. $Z_2 = 10 \, \text{kHz}$

(a) We want $A = 10$

At dc, $Z_1 = R_1$ and $Z_2 = R_2$ so $A = \frac{R_2}{R_1}$

$\Rightarrow$ need $R_2 = 10 \, \text{k}\Omega$

(b) We want $f_2 = 10 \, \text{kHz}$

At $f_2$, $R_1 = \frac{1}{2\pi f_2 C_1}$ so $C_1 = \frac{1}{2\pi f_2 R_1}$

$= \frac{1}{2\pi (10\,\text{kHz})(1\,\text{k}\Omega)}$

$= 0.016 \, \mu\text{F}$
we want $f_p = 100 \text{kHz}$

at $f_p$, $R_2 = \frac{1}{2\pi f_p C_2}$

so $C_2 = \frac{1}{2\pi f_p R_2}$

$= \frac{1}{2\pi (100 \text{kHz})(10k \Omega)}$

$= 160 \text{ pF}$

So the circuit is

A problem with this design!

We are trying to obtain more gain than the op amp is capable of producing.

The formula $G(s) = -\frac{R_2}{s}$ is true for ideal op amps with $\text{Gain} \to \infty$ (i.e., for $|K_{\text{ip}}|$ sufficiently large). But at high frequency, $\text{Gain}$ rolls off.
What really happens:

To find loop gain (or measure loop gain):

inject a voltage $v_2$ at output of op amp
given $v_x$, find $v_y$. Loop gain is $T(s) = \frac{v_y}{v_x}$

with $v_i = 0$ -

Analysis: with $v_i = 0$, $v^- = \frac{z_1}{z_1 + z_2} v_x$ and $-v_y = -G_{op} v^-$
so $v_y = \frac{z_1}{z_1 + z_2} G_{op} v_x$ and $T(s) = \frac{z_1}{z_1 + z_2} \frac{G_{op}}{s}$
The actual gain is

\[ G(s) = \frac{v_o}{v_i} = \left( -\frac{Z_2}{Z_1} \right) \left( \frac{T}{1+T} \right) \]

\[ \text{ideal gain} \quad \text{effect of finite gain} \]

Construct loop gain:

\[ \frac{Z_1}{Z_1 + Z_2} = \frac{1}{1 + \frac{Z_2}{Z_1}} \]

Since \( ||Z_2|| \gg ||Z_1|| \) everywhere, this is approximately

\[ \frac{1}{1 + \frac{Z_2}{Z_1}} \approx \frac{Z_1}{Z_2} \]

which is \( \frac{1}{\text{ideal gain}} \)

\[ \left| \frac{Z_1}{Z_2} \right| \]

So \( T \approx \frac{Z_1}{Z_2} \text{Gap} \) is decreased relative to Gap.
Find crossover frequency where $|T| = 1$

$$T = \frac{Z_1}{2 \pi f_2} C_{op} \approx \frac{Z_1}{2 \pi f_2} G_{op} = R_1 \frac{1 + \frac{s}{\omega_{op}}}{R_2 \left(1 + \frac{s}{\omega_2}\right)} \frac{G_o}{1 + \frac{s}{\omega_{op}}}$$

For $f_2 < f < f_p$, $T = R_1 \frac{1 + \frac{s}{\omega_{op}}}{R_2 \left(1 + \frac{s}{\omega_2}\right)} \frac{G_o}{1 + \frac{s}{\omega_{op}}}$

so $|T| = \frac{R_1}{R_2} G_o \frac{\omega_2 \omega_{op}}{\omega^2}$

At $f = f_c$, $|T| = 1 = \frac{R_1}{R_2} G_o \frac{f_2 f_{op}}{f_c^2}$

Thus $f_c^2 = \frac{R_1}{R_2} G_o f_2 f_{op}$

$$f_c = \sqrt{\frac{R_1}{R_2} G_o f_2 f_{op}} \approx 30 \text{ kHz}$$
What is the phase margin?

\[ \angle T \text{ at } f = f_c \text{ is} \]

-90° from the pole at \( f_p = 2 \text{kHz} \)

from the pole at \( f_2 \) and zero at \( f_p \):

see textbook Fig. 9.16 or Eq. (9.34).

The result is

\[ -\tan^{-1}\left(\frac{\sqrt{10} - \sqrt{0.1}}{2}\right) = -55° \]

\[ \angle T = -90° - 55° = -145° \text{ at } f_c \]

Phase margin = 180° + \( \angle T = 35° \)

From textbook Fig. 9.13, this leads to closed-loop Q of 1.57 \( \Rightarrow 4 \text{ dB} \)

\[ \left\| \frac{T}{1+T} \right\| \]

\[ \frac{T}{1+T} \approx \left\{ \begin{array}{ll} 1, & \left\| T \right\| \gg 1 \\ T, & \left\| T \right\| \leq 1 \end{array} \right. \]
Actual transfer function

\[- \frac{2Z}{Z_1} \frac{T}{1+T}\]

For \( f > f_c \),

\[ G \approx \left( -\frac{2Z}{Z_1} \right) \left( \frac{T}{1+T} \right) \]

\[ = \left( -\frac{2Z}{Z_1} \right) \left( \frac{T}{2Z} \right) \]

\[ = - \frac{2Z}{Z_1} T \]

So the actual transfer function follows the ideal value \((-\frac{2Z}{Z_1})\) as long as \( ||G_{op}|| > \frac{2Z}{Z_1} || \). But when \( ||G_{op}|| < \frac{2Z}{Z_1} || \), then the actual transfer function follows \((-G_{op})\) instead.