Design of op amp PID compensator circuit for the example of Section 9.5.4

We want to synthesize an op amp circuit that realizes the feedback block diagram below (excerpted from Fig. 9.22):

\[ G_c(s) = G_{oo} \frac{1 + \frac{s}{\omega_2}}{1 + \frac{s}{\omega_p}} \]

(From Eq. (9.60))

\[ G_{oo} = 3.7 \Rightarrow 11.3 \text{ dB} \] is the midband gain shown above (why?)
We need a candidate op amp circuit that can realize all of the functions of block diagram. Here are some basic circuits from the beginning circuits class.

Voltage divider - suitable for \( H \)

\[
H = \frac{R_{2d}}{R_{1} + R_{2d}}
\]

Subtractor - suitable for summing node

if we let \( Z_1 = Z_2 \) then

\[ v_o = v^+ - v^- \]

we could use this circuit with

\[ v^- \rightarrow H v^- \]

\[ v^+ \rightarrow \hat{v}_{ref} \]

\[ v_o \rightarrow \hat{v}_e \]

More generally, with \( Z_1 \neq Z_2 \), we get

\[ v_o = \frac{Z_2}{Z_1} (v^+ - v^-) \]

By proper choice of \( \frac{Z_2}{Z_1} = C_c(s) \), we could use

\[ v^- \rightarrow H v^- \]

\[ v^+ \rightarrow \hat{v}_{ref} \]

and \( v_o \rightarrow \hat{v}_c \)
we need to synthesize series-parallel combinations of \( R \)'s and \( C \)'s so that \( \frac{Z_2}{Z_1} \) has the \( G_c(s) \) asymptotes of page 1.

we can get \( G_c \) asymptotes that have the above shapes by placing the above impedances in \( Z_2 \).

If we put the above impedance combinations in \( Z_1 \), then \( G_c \) will have asymptotes proportional to \( \frac{1}{Z_1} \) that look like

the \( G_c \) we want:

we will add another series resistor in \( Z_1 \) to make its asymptotes flatten out - see below
So, \( Z_2 = R_2 + \frac{1}{5C_2} \)

\[ \frac{1}{\omega C_2} \]

\[ R_2 \]

\[ Z_2 \]

\[ L \]

\[ f_L \]

\[ R_2 = \frac{1}{2\pi f_L C_2} \]

\[ Z_1 = (R_1 || \frac{1}{5C_1}) + R_3 \]

\[ R_1 \]

\[ \frac{1}{\omega C_1} \]

\[ R_3 \]

\[ f_2 \]

\[ \frac{1}{\omega C_1} \]

\[ f_3 \]

\[ R_1 = \frac{1}{2\pi f_C C_1} \]

\[ R_3 = \frac{1}{2\pi f_p C_1} \]

Construct \( G_c = \frac{Z_2}{Z_1} \)

Divide asymptotes

\[ \frac{1}{\omega C_2} \]

\[ f_L \]

\[ f_2 \]

\[ f_3 \]

\[ Z_1 = R_1 \]

\[ Z_2 = R_2 \]

\[ Z_3 \]

\[ G_c = \frac{R_2}{R_3} \]
So the op amp circuit is

Choose element values
from above:

\[ G_{\text{co}} = \frac{R_2}{R_1} \quad R_2 = \frac{1}{2\pi f_L C_2} \]

\[ R_1 = \frac{1}{2\pi f_L C_1} \quad R_3 = \frac{1}{2\pi f_P C_1} \]

We have 4 equations and 5 unknowns.

One element value can be chosen arbitrarily, and this choice determines the impedance and current levels in the circuit.

Choose \( R_2 = 100 \text{ k}\Omega \) to get currents of \( 10^3 \text{ } \mu\text{A} \)
then \( C_2 = \frac{1}{2\pi f_L R_2} = 3.2 \text{ nF} \) (with \( f_L = 500 \text{ Hz} \))

and \( R_1 = \frac{R_2}{G_{\text{co}}} = \frac{100}{3.7} = 27 \text{ k}\Omega \)

\[ C_1 = \frac{1}{2\pi f_R R_1} = 3.5 \text{ nF} \] with \( f_R = 1.7 \text{ kHz} \)

\[ R_3 = \frac{1}{2\pi f_P C_1} = 3.3 \text{ k}\Omega \] with \( f_P = 14 \text{ kHz} \)
Some additional considerations:

1. In this dc regulator application, if $V_{\text{ref}}(t) = V_{\text{ref}}$ is fixed (so $V_{\text{ref}} = 0$ always) then $V^+ = V_{\text{ref}}$ never changes, and we could simply connect $V_{\text{ref}}$ directly to $V^+$:

![Diagram showing a control circuit with input $V_{\text{ref}}$ and output $V_c$](image)

This will, however, add poles and zeros to the transfer function from $V_{\text{ref}}$ to $V_c$. So if $V_{\text{ref}}(t)$ can have a significant ac component, then it would be desirable to include $Z_1$ and $Z_2$ in the $V_{\text{ref}}$ circuit.

In the above circuit: if the op amp has a significant input bias current, then it may be desirable to insert a resistor of value $R_1 + R_3$ between $V_{\text{ref}}$ and the noninverting input so that the bias current does not affect the regulated dc output voltage.
2. When we connect this op amp circuit to the \( H(s) \) voltage divider, \( Z_1 \) loads the divider:

![Circuit Diagram]

We can account for this in our design; the series equivalent of the voltage divider:

\[ R_{th} = \frac{R_{1d} || R_2}{R_{1d} + R_2} \]

\[ R_{th} = R_{1d} || R_2 \text{ is effectively in series with } Z_1 \]

If we choose \( R_{1d} || R_2 = R_{th} \) to be equal to \( R_3 \), then we could eliminate \( R_3 \) and use \( R_{th} \) instead:
to get this to work, we must choose
\[ R_{1d} || R_{2d} = R_3 = 3.3\, \text{k} \]

with \[ \frac{R_{2d}}{R_{1d} + R_{2d}} = H = \frac{1}{3} \]

note that \[ R_{1d} || R_{2d} = H \cdot R_{1d} \]
so we need \[ R_{1d} = \frac{R_3}{H} = 10\, \text{k} \]
and \[ R_{2d} = R_{1d} \frac{H}{1-H} = 5\, \text{k} \]

3. This design asks the op amp to have gain
but to infinite frequency. This won't happen
with a practical, physical op amp. Our \( G_c \) is
\[ \frac{G_c}{f_c} = \frac{3.7}{14\, \text{kHz}} \]
\[ G_c = 3.7 \implies f_c = 500 \, \text{Hz} \]
\[ f_p = 1.7\, \text{kHz} \]
Suppose our op amp has a gain-bandwidth product of 1 MHz, so that its internal gain is

\[ |\text{Gain}| = \frac{1}{\left(\frac{f}{f_{\text{GBP}}}\right)} \text{ with } f_{\text{GBP}} = 1 \text{ MHz} \]

This asymptote is of the form

\[ |\text{Gain}| = \frac{1}{\left(\frac{f}{f_{\text{GBP}}}\right)} \]

It has the value 30.5 at frequency \( f_a \):

\[ 30.5 = \frac{1}{\left(\frac{f_a}{f_{\text{GBP}}}\right)} = \frac{f_{\text{GBP}}}{30.5} = 32.8 \text{ kHz} \]

Above \( f_a \), the op amp is not capable of producing the desired gain, so the \( G_c \) transfer function will roll off:

The added pole at \( f_a \) will degrade the phase margin, so we will need to account for it in our design.