Middlebrook’s Extra Element Theorem

The effect of capacitor equivalent series resistance (esr) on the transfer function of an R-L-C filter. In the simple R-L-C low-pass filter illustrated in Fig. 1 below, the capacitor dielectric loss, contact resistance, and foil resistance are modeled by a series resistance known as the capacitor esr. Physical capacitors often contain significant esr, which can degrade performance and can also lead to capacitor failure when the power loss \( I^2 \text{esr} \) causes excessive temperature rise within the capacitor. The objective of this short problem is to derive conditions which ensure that the esr does not significantly affect the filter transfer function.

(a) Use the extra element theorem to derive conditions that ensure that the esr does not significantly modify the transfer function \( G(s) = \frac{v_2(s)}{v_1(s)} \).

(b) For the values \( L = 100 \) µH, \( C = 1 \) µF, \( R = 100 \) Ω, esr = 2 Ω, sketch the Bode plots of your impedance conditions (i.e., the \( Z_N \) and \( Z_D \) magnitudes) derived in Part (a). Overlay the value of the esr. Over what ranges of frequencies does the esr significantly change the transfer function? In what ways is the transfer function changed?

Analysis of the CCM buck-boost converter control-to-output transfer function \( G_{vd}(s) \) using the extra element theorem. Averaged switch modeling of the CCM buck-boost converter leads to the small-signal ac model illustrated in Fig. 2. One approach to solving for \( G_{vd}(s) \) in this circuit is to employ the extra element theorem, treating inductor \( L \) as the extra element.

(a) Let \( L \rightarrow \) short circuit, and determine the “original transfer function” \( G_{d0} \).

(b) Determine \( Z_N(s) \) and \( Z_D(s) \), and hence derive the expression for \( G_{vd}(s) \).

Note: no credit will be given for methods that do not employ the extra element theorem.