MIMICAD TECHNICAL REPORT NO. 6

Multiport Network Model for Evaluating Radiation Loss and Spurious Coupling among Discontinuities in Microstrip Circuits

by

Albert Sabban and K.C. Gupta

Department of Electrical and Computer Engineering
University of Colorado
Boulder, Colorado 80309-0425

Work described in this report has been supported by the Center for Microwave/Millimeter-Wave Computer-Aided Design

January 1991
ABSTRACT

A multiport network model for evaluating radiation loss and spurious coupling among discontinuities in microstrip circuits is proposed in this report. A planar multiport network model of the discontinuity configuration and the segmentation method are used to evaluate voltage distribution around the edges of the discontinuity. This voltage distribution is expressed as an equivalent magnetic current line source distribution which is used to calculate the far-zone field (for radiation loss) or the near-zone field (for spurious coupling calculation).

In this report we have developed a planar-lumped model for characterization of several of the coupled-line junction configurations. The planar-lumped model is a combination of two-dimensional planar and lumped-element networks. Fields underneath the two strips and those fringing at the outer edges are modeled by planar waveguides. The electric field and magnetic field couplings across the gap are represented by equivalent capacitive and inductive lumped networks. Model verification for coupled line sections for the dominant and higher order modes is reported in this report.

The planar-lumped model is used to evaluate radiation loss and spurious coupling among coupled microstrip line discontinuities.

Modal expansion of fields and the multiport network model are used to investigate the effect of enclosures on parasitic coupling between microstrip discontinuities.
ACKNOWLEDGEMENTS

This report is based on the Ph.D. Thesis submitted by Albert Sabban in January 1991.
# CONTENTS

## CHAPTER

### I. INTRODUCTION

1.1 Background ......................................................... 1

1.2 Previous Work on Radiation and Spurious Coupling
   Caused by Discontinuities............................................. 2

1.3 Approach Used in the Present Investigation ...................... 5

1.3.1 Two-Dimensional Planar Model ................................. 5

1.3.2 Multiport Network Model ........................................ 7

1.3.3 Segmentation Method ............................................. 8

1.4 Organization of the Report ....................................... 8

### II. RADIATION LOSS FROM MICROSTRIP DISCONTINUITIES .......................... 10

2.1 The Approach ..................................................... 10

2.2 Planar Model for Microstrip Discontinuities .................... 10

2.3 Multiport Model for Microstrip Discontinuities ................ 13

2.3.1 Fields Underneath the Microstrip Circuit .................... 13

2.3.2 External Fields ................................................ 14

2.3.3 MNM for Evaluating Edge Voltages ............................ 16

2.4 Radiation Fields of a Magnetic Current Element ............... 17

2.5 Radiation Fields of a Magnetic Current Line Source ........... 20

2.6 Radiated Power from Microstrip Discontinuities ................ 21
<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.7 Incorporation of Radiation Loss in S-Parameters of Discontinuities</td>
<td>23</td>
</tr>
<tr>
<td>2.7.1 Sample Results for S-Matrix</td>
<td>23</td>
</tr>
<tr>
<td>2.8 Approximations Involved in MNM Approach</td>
<td>23</td>
</tr>
<tr>
<td>2.8.1 Two-Dimensional Fields</td>
<td>23</td>
</tr>
<tr>
<td>2.8.2 Matching the Fields at Interconnecting Ports</td>
<td>26</td>
</tr>
<tr>
<td>2.8.3 Surface Waves Are Not Accounted for in the Model</td>
<td>27</td>
</tr>
<tr>
<td>2.8.4 Radiation Fields of a Magnetic Current Element</td>
<td>27</td>
</tr>
<tr>
<td>III. NUMERICAL RESULTS FOR RADIATION LOSS FROM MICROSTRIP DISCONTINUITIES</td>
<td>31</td>
</tr>
<tr>
<td>3.1 Introduction</td>
<td>31</td>
</tr>
<tr>
<td>3.2 Computational Details</td>
<td>31</td>
</tr>
<tr>
<td>3.2.1 Number of Terms in Green's Function Expansion</td>
<td>32</td>
</tr>
<tr>
<td>3.2.2 Port Width</td>
<td>33</td>
</tr>
<tr>
<td>3.2.3 Radiating Region of a Discontinuity Configuration</td>
<td>38</td>
</tr>
<tr>
<td>3.3 Voltage Distribution Along the Edges of Microstrip Discontinuities</td>
<td>39</td>
</tr>
<tr>
<td>3.3.1 Chamfered Right-Angled Bend</td>
<td>39</td>
</tr>
<tr>
<td>3.3.2 Right-Angled Bend (Un chamfered) on GaAs Substrate</td>
<td>40</td>
</tr>
<tr>
<td>3.3.3 Right-Angled Bend (Un chamfered) on 0.254 mm thick Duroid Substrate</td>
<td>40</td>
</tr>
<tr>
<td>3.3.4 Step Junction</td>
<td>47</td>
</tr>
<tr>
<td>3.3.5 Stub</td>
<td>47</td>
</tr>
<tr>
<td>3.4 Radiation from a Right-Angled Microstrip Bend</td>
<td>49</td>
</tr>
<tr>
<td>3.5 Radiation from a Chamfered Bend</td>
<td>49</td>
</tr>
<tr>
<td>3.6 Radiation from a Compensated Bend with a Change in Line Impedance</td>
<td>51</td>
</tr>
<tr>
<td>3.7 Radiation from a Microstrip Step Junction</td>
<td>54</td>
</tr>
</tbody>
</table>
3.8 Radiation from a T-junction Discontinuity ...................................... 56
3.9 Radiation from a 20Ω Stub .............................................................. 57
3.10 Radiation from a Gap Discontinuity .................................................. 58
3.11 Radiation from Asymmetric Step Discontinuity when Higher-Order Modes are Excited .............................................................. 60
3.12 Comparison of Power Radiated Results With Fullwave Analysis Results .............................................................. 62

IV. EXPERIMENTS TO VERIFY RADIATION FROM MICROSTRIP BENDS .............................................................. 74
4.1 Design of Experiment ................................................................. 74
4.2 Comparison of Measured and Calculated Resonant Frequencies ................. 75
4.3 Computation of Q-Factors ............................................................ 78
4.4 Measurements of Q-Factors ......................................................... 80
4.5 Computation of Radiation Q-Factors Accounting for Measured Losses in Linear Resonators .................................................. 81

V. EVALUATION OF SPURIOUS COUPLING AMONG DISCONTINUITIES IN MICROSTRIP CIRCUITS .............................................................. 84
5.1 Approach .................................................................................... 84
5.2 Microstrip Edge Field and Associated Magnetic Current Distribution .............. 85
5.3 Interaction between Magnetic Currents and Mutual Coupling ......................... 88
5.3.1 Linear Distribution of Magnetic Current ....................................... 89
5.3.2 Two-Dimensional Distribution of Magnetic Current ...................... 90
5.4 Coupling Calculation ................................................................. 96
5.5 Verification of Coupling Calculation ............................................. 98
VI. EXPERIMENTAL VERIFICATIONS OF THE MULTIPORT
NETWORK MODEL FOR EVALUATING SPURIOUS
COUPLING AMONG DISCONTINUITIES IN
MICROSTRIP CIRCUITS........................................... 101

6.1 Two Configurations Used for Experimental Verification. 101
6.2 Interaction between Two Open-Ended Stubs............... 101
6.2.1 Configuration............................................. 101
6.2.2 Voltage Distribution Around the Edges of the Double
Stub Circuit....................................................... 102
6.2.3 Effect of Radiation Loss on Transmission Coefficient... 102
6.2.4 Effect of Parasitic Coupling on S-parameters............. 106
6.3 Coupling between Two Parallel Lines with Step
Discontinuity and Comparison to Measured Results......... 113

VII. MNM RESULTS FOR SPURIOUS COUPLING................. 117

7.1 Computational Details for MNM Approach for
Evaluation of Spurious Coupling............................... 117
7.2 Coupling between Two Right-Angled Bends................. 119
7.3 Coupling between Two Open-Ends.......................... 123
7.4 Comparison of Coupling Results with Fullwave Analysis
Results............................................................... 126
7.5 Coupling among Three Right-Angled Bends................. 128

VIII. EXTENSION OF MNM METHOD FOR COUPLED LINE
DISCONTINUITIES.................................................... 132

8.1 Introduction.................................................. 132
8.2 A Planar-Lumped Model for Coupled Microstrip Lines... 133
8.2.1 Planar Waveguides Parameters.......................... 135
8.2.2 C-Network for Modeling E-Field Coupling.............. 136
8.2.3 L-Network for Modeling H-Field Coupling............... 137
8.3 Model Verification for Dominant Mode...................... 143
8.4 Model Verification for Higher-Order Modes............... 145
8.5 Comparison with Experimental Results for a Coupled Line Section with Chamfered Bends 155

8.6 Discontinuity Effects in a Single Section Coupler 158

8.6.1 A Directional Coupler with Four Right-Angled Bends 161

8.6.2 A Coupler with Four Chamfered Bends 162

8.6.3 Discontinuity Effects in a 10 dB and 25 dB Couplers 165

8.6.4 Comparison with Fullwave Analysis for a Coupler with Four Chamfered Bends 175

8.7 Discontinuity Effects in a Three-Section Coupler 176

IX. RADIATION LOSS AND PARASITIC COUPLING CAUSED BY COUPLED MICROSTRIP LINE DISCONTINUITIES 183

9.1 Introduction 183

9.2 Evaluation of Radiation Loss from Coupled Microstrip Line Discontinuities 184

9.3 Power Radiated From a Coupled Line with Four Chamfered Bends 185

9.4 Parasitic Coupling Caused by Coupled Microstrip Line Discontinuities 188

9.5 Computational Details for Evaluation of Spurious Coupling between Coupled Microstrip Line Discontinuities 190

9.6 Parasitic Coupling between a Coupled Line Section with Two Right-Angled Bends 192

9.7 Spurious Coupling between a Coupler with Four Chamfered Bends and a Right-Angled Bend 192

9.8 Spurious Coupling between a Coupler with Four Chamfered Bends and a Chamfered Bend for Different Orientations of the Chamfered Bend 196

X. THE EFFECTS OF ENCLOSURES ON PARASITIC COUPLING BETWEEN MICROSTRIP DISCONTINUITIES 201

10.1 Package Effects on Microstrip Performance 201

10.2 Method of Analysis 202
10.3 Computational Details for Calculation of Spurious Coupling in an Enclosure .................................................. 207

10.3.1 Convergence of Coupling Results as a Function of the Number of Modes in the Enclosure .......................... 208

10.3.2 Convergence of Coupling Results between Two Open Ends in an Enclosure ............................................. 209

10.3.3 Convergence of Coupling Results between Two Right-Angled Bends in an Enclosure .................................. 209

10.4 Examples of Parasitic Coupling Evaluation in the Presence of a Rectangular Enclosure .............................. 212

10.4.1 Coupling between Two Colinear Open Ends in a Metal Enclosure ......................................................... 212

10.4.2 Parasitic Coupling between Two Corners of Right-Angled Bends as a Function of Enclosure Dimensions ... 213

XI. CONCLUDING REMARKS ........................................... 218

11 Conclusions .......................................................... 218

11.1 Summary of Salient Results ..................................... 218

11.1.1 Multiport Network Model for Evaluating Radiation Loss from Microstrip Discontinuities ....................... 218

11.1.2 Multiport Network Model for Evaluating Spurious Coupling between Discontinuities in Microstrip Circuits .... 219

11.3 Planar-Lumped Model for Coupled Microstrip Line Discontinuities ....................................................... 220

11.1.4 The Effect of Enclosures on Parasitic Coupling between Microstrip discontinuities ................................ 221

11.2 Suggestions for Further Work ................................ 222

11.2.1 Incorporating Radiation Loss and Spurious Coupling among Discontinuities in Microstrip Antenna Arrays .... 222

11.2.2 Evaluation of Radiation Loss and Spurious Coupling among Discontinuities in Microstrip Circuits Covered with a Dielectric Layer ....................................................... 222

11.2.3 Improved MNM for Evaluation of Spurious Coupling in an Enclosure ....................................................... 223
REFERENCES ................................................................. xi

APPENDIX

A. GREEN FUNCTION APPROACH FOR COMPUTATION OF Z- MATRICES OF PLANAR SEGMENTS .............................................. 230
   A.1 Z-Matrix of Planar Network with Magnetic Boundary ... 230
   A.2 Computation of Green’s Function .................................. 233
   A.3 Green’s function for a rectangular planar segment .......... 233
B. SEGMENTATION METHOD .................................................. 235
C. SPECTRAL DOMAIN ANALYSIS FOR EVALUATION OF MICROSTRIP LINE PARAMETERS ...................................................... 237
   C.1 Introduction ........................................................... 237
   C.2 Formulation of the Problem .......................................... 238
   C.3 Computations of Characteristic Impedance for Microstrip Lines ................................................................. 242
   C.4 Results for Effective Dielectric Constant and Characteristic Impedance Using Spectral Domain Method ...... 245
   C.5 Higher-Order Modes in Microstrip Lines ......................... 248
D. SPECTRAL DOMAIN ANALYSIS FOR EVALUATION OF COUPLED MICROSTRIP LINE PARAMETERS ................................. 253
   D.1 Approach Used ........................................................ 253
   D.2 Numerical Results for Coupled Microstrip Lines ............. 256
E. DERIVATION OF CPAACITANCE AND INDUCTANCE MATRICES FOR THE PLANAR-LUMPED MODEL FOR COUPLED MICROSTRIPS ................................................................. 261
   E.1 Derivation of Capacitance [C] Matrix ............................. 261
   E.2 Derivation of Inductance Matrix ..................................... 264
F. CONVERGENCE OF S-PARAMETERS FOR THE PLANAR-LUMPED MODEL OF A COUPLED LINE SECTION AS A FUNCTION OF THE NUMBER OF PORTS IN THE GAP REGION ......................... 272
FIGURES

FIGURE

2.1 Planar waveguide model for microstrip line. ..................................... 11
2.2 Planar model for microstrip bends. .................................................... 12
2.3 Multiple ports located at the edges of the microstrip discontinuities. .......... 16
2.4 Equivalent magnetic current distribution at discontinuity edges. .......... 18
2.5 Fields produced by a magnetic current element. ................................. 19
2.6 A magnetic line source and coordinate system for evaluating radiation fields. ................................................................. 20
2.7 Coordinate system for external field calculations. ............................. 22
2.8 Multiport network model for incorporating radiation loss in S-parameters. .................................................................................. 24
2.9 Magnetic current modeling of edge fields. .......................................... 30

3.1(a) Typical voltage distribution at the edges of a microstrip chamfered bend (voltage amplitude in volts). $\varepsilon_r = 2.2$; $h = 0.1$ inch; $Z_0 = 50\Omega$; $f = 10$ GHz. ................................................................. 41

3.1(b) Phase distribution of the edge voltages for a chamfered bend (phase values shown in degrees). $\varepsilon_r = 2.2$; $h = 0.01$ inch; $Z_0 = 50\Omega$; $f = 10$ GHz. ................................................................. 42

3.2(a) Typical voltage distribution at the edges of a microstrip right-angled bend (voltage amplitude in volts). $\varepsilon_r = 12.9$; $h = 0.127$ millimeter; $Z_0 = 50\Omega$; $f = 10$ GHz. ................................................................. 43

3.2(b) Phase distribution of the edge voltages for a right-angled bend (Phase values shown in degrees). $\varepsilon_r = 12.9$; $h = 0.1271$ millimeter; $Z_0 = 50\Omega$; $f = 10$ GHz. ................................................................. 44
3.2(c) Typical voltage distribution at the edges of a microstrip right-angled bend (voltage amplitude in volts) \( \varepsilon_r = 2.2; \ h = 10 \text{ mil}; \ Z_0 = 50 \Omega; \ f = 10 \text{ GHz}. \) ......................................................... 45

3.2(d) Phase distribution of the edge voltage for a right-angled bend (phase values in degrees) \( \varepsilon_r = 2.2; \ h = 10 \text{ mil}; \ Z_0 = 50 \Omega; \ f = 10 \text{ GHz}. \) ......................................................... 46

3.3 Voltage distribution at the edges of a step junction input power is fed from a 50\( \Omega \) line. \( \varepsilon_r = 2.2; \ h = 0.01 \text{ mil}; \ Z_1 = 70.7 \Omega; \ Z_2 = 70.7 \Omega \ f = 10 \text{ GHz}. \) Voltage magnitude in volts, phase angle in degrees. ......................................................... 48

3.4 Voltage distribution at the edges of a 20\( \Omega \) open-end stub. .............. 50

3.5 Normalized radiated power from a right-angled bend (MNM: Multiport Network Model and PV: Poynting Vector Method). ................................................................. 51

3.6 Radiated power from bend discontinuity on GaAs substrate. ............ 52

3.7 Multiple ports along the edges of a chamfered bend discontinuity. ...... 53

3.8 Power radiated from a chamfered bend. .................................................. 54

3.9 Multiport network modeling of a compensated bend. ....................... 55

3.10 Radiated power from a compensated bend on substrate with \( \varepsilon_r = 2.2, 10 \text{ mil thick}. \) ................................................................. 55

3.11 Normalized radiated power from a step (MNM: Multiport Network Model and PV: Poynting Vector Method). ................................................................. 56

3.12 Normalized radiated power from a T-junction.(MNM: Multiport Network Model and PV: Poynting Vector Method). ......................................................... 57

3.13 Normalized radiated power from a 20\( \Omega \) open-end stub (MNM: Multiport Network Model and Poynting Vector Method). ......................................................... 58

3.14 \( \pi \)-network representation of the \([C]\) matrix. ........................................... 59

3.15 Multiport network model for a gap discontinuity. ......................... 61

3.16 Power radiated from asymmetric gap. .................................................. 63

3.17 Multiple ports for an asymmetric step discontinuity from 50\( \Omega \) to 20\( \Omega \). ................................................................. 64

3.18 Power radiated from asymmetric step when higher-order modes are excited. ......................................................... 65
3.19 Comparison of power radiated results from an open end on \( \varepsilon_r = 2.2 \) substrate, with P-Mesh and PV Method. .......... 72

3.20 Comparison of power radiated from a 50\( \Omega \) bend on \( \varepsilon_r = 2.2 \) substrate, with PV results and fullwave analysis results. .......... 73

4.1 Gap coupled resonant structures. ............................................... 76

5.1 Network model for evaluating spurious coupling between discontinuities. ....................................................... 85

5.2 Edge voltage distribution. .......................................................... 87

5.3 A two-port network. ..................................................................... 91

5.4 Coupling between two apertures. .................................................. 95

5.5 Equivalent magnetic current distribution. ...................................... 97

5.6 Comparison of \( S_{12} \) for parallel coupled lines. ......................... 100

6.1 Configuration of the double-stub circuit. ....................................... 103

6.2(a) Edge voltages (in volts) at various points along the double-stub circuit. .......................................................... 104

6.2(b) Phase distribution of the edge voltages for the double-stub circuit. ........................................................................... 105

6.3 \( S_{22} \) results near resonance of the two stubs. Curve 3 considers isolated discontinuities, while curve 1 accounts for internal interaction. ................................................... 107

6.4 MCN for the double-stub circuit. ..................................................... 109

6.5 Comparison of \( S_{12} \) (in dB) obtained from MNM approach with fullwave and measured results. .................................................. 110

6.6 Comparison of \( S_{12} \) (in dBs) obtained from MNM approach with P-Mesh Code. ................................................................. 111

6.7 Comparison of MNM approach results for \( S_{11} \) (in dBs) with fullwave results and measured results. .................................................. 112

6.8 Partial coupling between different parts of the double-stub circuit. .................................................. 114

6.9 Two parallel lines with step discontinuity. ..................................... 115

6.10 Multiport network representation of coupling between two circuit components. .......................................................... 116
7.1 Coupling between corner of a right-angled bend as a function of spacing $d$ (in mm). .......................................................... 120
7.2 $S_{13}$ results between $50\Omega$ bends on substrate with $\varepsilon_r = 2.2$ 10 mil thick, $L = 1.055cm$. ........................................ 121
7.3 Convergence of $S_{13}$ as a function of the distribution width. .................. 122
7.4 Coupling results $S_{13}$ at 40 GHz between two corners of right-angled bends. ............................................................................. 124
7.5 $S_{13}$ results between two $50\Omega$ right-angled bends on $\varepsilon_r = 2.2$ substrate, 10 mil thick, $L = 5.427mm$. .................................. 125
7.6 Coupling results $S_{12}$ between two open ends. ...................................... 126
7.7 Comparison of $S_{12}$ results between two open ends on $\varepsilon_r = 10.5$ substrate, $1.27mm$ thick. ......................................................... 127
7.8 Comparison of $S_{12}$ results between two open ends with P-Mesh code results. ......................................................................................... 129
7.9 Coupling results between three corners of right-angled bends. ................. 131
8.1 A coupled line section and its planar-lumped model. ................................. 134
8.2 C-network for modeling E-field Coupling. ............................................... 138
8.3 L-network for modeling H-field coupling. ............................................... 140
8.4 Inductive part of the complete lumped network. ...................................... 142
8.5 Modified L-network. ............................................................................. 144
8.6 Comparison of coupling computed by planar/lumped model with conventional coupled line analysis. ......................................................... 147
8.7 Planar waveguide model of a single microstrip line and its equivalence in the transverse direction. ......................................................... 148
8.8 Planar-lumped model to evaluate transverse resonance. ............................ 149
8.9 Y-network representation of mutual inductance element. ......................... 151
8.10 Equivalent representation for planar-lumped model to evaluate higher-order modes. ........................................................................ 152
8.11 Network representation in the transverse direction for even and odd modes. ......................................................................................... 153
8.12(a) A coupled line with chamfered bends. .............................................. 156
8.12(b) Planar-lumped model for coupled line configuration of Figure 8.7.

8.13(a) Comparison of $S_{12}$ values for coupled line configuration of Figure 8.12 as obtained by: (1) planar model, (2) conventional coupled line analysis, and (3) measurements.

8.13(b) Comparison of $S_{12}$ values for coupled line configuration of Figure 8.12 as obtained by: (1) planar model, (2) conventional coupled line analysis, and (3) measurements, all for spacing of 20 $\mu$m.

8.14 Comparison of $S_{12}$ values for coupled line with four 90° bends and an ideal coupled lines section.

8.15 Comparison of $S_{13}$ values for an ideal coupled line section and a coupled line section with four right-angled bends.

8.16 Comparison of $S_{11}$ values in dB for an ideal coupled line section and a coupled line with four 90° bends.

8.17 Comparison of coupling values for the three configurations shown.

8.18 Comparison of $S_{12}$ in dB for a 10 dB coupler for the three configurations shown.

8.19 Comparison of $S_{12}$ in dB for a 25 dB coupler for the three configurations shown.

8.20 Comparison of $S_{12}$ in dB for the three configurations shown, 10 dB couplers.

8.21 Comparison of $S_{12}$ in dB for the three configurations shown, 25 dB couplers.

8.22 Comparison of $S_{14}$ phase in degrees ($\lambda/4$ between center line of bends), for 10 dB couplers.

8.23 Comparison of $S_{14}$ phase in degrees ($\lambda/4$ between center line of bends) for 25 dB couplers.

8.24 Comparison of $S_{14}$ ($\lambda/4$ between ends of bends) for the three configurations shown (phase evaluated with respect to the four reference planes shown in Figure 8.23), for 10 dB couplers.

8.25 Comparison of $S_{14}$ ($\lambda/4$ between ends of bends) for the three configurations shown (phase evaluated with respect to the four reference planes shown), for 25 dB couplers.

8.26 Comparison of $S_{12}$ results with fullwave analysis for a coupler with four chamfered bends.
8.27 A 10 dB three-section coupler. ........................................ 179
8.28 Comparison of $S_{12}$ results for a three-section coupler. ................ 180
8.29 Comparison of $S_{11}$ results for a three-section coupler. ................ 181
8.30 Comparison of $S_{14}$ results for a three-section coupler. ................ 182
9.1 Multiple ports located at the edges of a coupled microstrip line discontinuity. ........................................ 186
9.2 Radiated power from a coupled line with four chamfered bends. ........ 187
9.3 Radiated power from a coupled line section with four chamfered bends as a function of spacing. ...................... 188
9.4 Network model for evaluating coupling between microstrip discontinuities associated with coupled line sections. .......... 189
9.5 Multiport network model for evaluating parasitic coupling between a single microstrip bend and a coupled microstrip discontinuity. ........................................ 190
9.6 Multiport network model for incorporating coupling and electrostatic coupling between two bends connected to a coupled line section. ...................... 193
9.7 $S_{13}$ for a coupled section with two right-angled bends. .................. 194
9.8 Effect of a right-angled bend in proximity on coupled $S_{21}$ ......... 195
9.9 $S_{13}$ results between a coupler and a right-angled bend. .............. 197
9.10 $S_{12}$ results between a coupler and a chamfered bend. .................. 198
9.11 $S_{13}$ results between a coupler and a chamfered bend. .................. 200
10.1 Equivalent magnetic current modeling of two bends in an enclosed microstrip circuit. ........................................ 203
10.2 A magnetic current element source in a rectangular enclosure. .......... 204
10.3 Circuit with equivalent network model of parasitic coupling in an enclosed environment. ........................................ 205
10.4 A magnetic current element source in a rectangular enclosure with no side walls. ........................................ 208
10.5 Comparison of $S_{12}$ results between two open ends with Em program results and moment method results, $Y = 24\,\text{mm}$. .... 210
10.6 Comparison of $S_{12}$ results between two open ends with EM program results and moment method results, $Y = 12\text{mm}$. ........ 214

10.7 $S_{13}$ results between two right-angled bends in an enclosure, $H = 1\text{cm}$. ......................................................... 216

10.8 $S_{13}$ results between two right-angled bends in an enclosure with $H = 3\text{cm}$. ......................................................... 217

A.1 A microstrip component excited by a $Z$-directed electric current source at $(X_0, Y_0)$. ......................................................... 231

C.1 Open microstrip resonant structure. ......................................................... 238

C.2 Spectral domain equivalent circuit. ......................................................... 240

C.3 Effective dielectric constant for a microstrip line on $\varepsilon_r = 9.9$, $h = 0.64\text{mm}$, $t = 5\mu\text{m}$, $w = 0.5\text{mm}$. ......................................................... 247

C.4 Comparison of computed characteristic impedance for a microstrip line on $\varepsilon_r = 9.9$, $h = 0.64\text{mm}$, $t = 5\mu\text{m}$, $w = 0.5\text{mm}$. ......................................................... 248

C.5 Comparison of effective dielectric constant results for a $0.785\text{mm}$ wide microstrip line on $\varepsilon_r = 2.2$ substrate, 10 mil thick. ......................................................... 249

C.6 Comparison of characteristic impedance results of a $0.785\text{mm}$ wide microstrip line on $\varepsilon_r = 2.2$ substrate, 10 mil thick. ......................................................... 250

C.7 Comparison of effective dielectric constant for first-order mode with Jansen and Kompa results. ......................................................... 251

C.8 Comparison of effective dielectric constant results for a $9.15\text{mm}$ wide microstrip line on $\varepsilon_r = 9.7$ substrate, $0.635\text{mm}$ thick, for the first and second higher-order mode. ......................................................... 252

D.1 Current basis functions for the microstrip line. ......................................................... 254

D.2 Current distribution for coupled microstrip lines. ......................................................... 255

D.3 Even and odd mode effective dielectric constant for a coupled line section on $\varepsilon = 9.9$ substrate, $0.635\text{mm}$ thick. ......................................................... 257

D.4 Comparison of $Z_{oe}$ and $Z_{00}$ for a coupler on $\varepsilon_r = 9.9$ substrate, $h = 0.635\text{mm}$ thick. ......................................................... 258

D.5 Even and odd mode effective dielectric constant for a coupler on $\varepsilon_r = 2.2$ substrate, 10 mil thick. ......................................................... 259

D.6 Even and odd mode characteristic impedance for a coupler on $\varepsilon_r = 2.2$ substrate, 10 mil thick. ......................................................... 260
TABLES

**TABLE**

2.1 Effect of Radiation on SWR and S-parameters .................................. 25

3.1 Comparison of double and single infinite series for computation of elements of the Z-matrix for a rectangular element........................................ 32

3.2 Comparison of double and single infinite series for computation of the elements of the Z-matrix for an isosceles triangular segment... 33

3.3 Edge voltages for a 50Ω microstrip line 0.8λ long at 10 GHz on GaAs substrate 0.127mm thick with ε_r = 12.9, for port width of 0.08λ (10 ports along each edge.) ............................................. 35

3.4 Edge voltages for a 50Ω microstrip line 0.8λ long at 10 GHz on GaAs substrate 0.127mm thick with ε_r = 12.9. The port width is 0.057λ (14 ports along each edge). .................................................. 36

3.5 Edge voltages for a 50Ω microstrip line 0.8λ long at 10 GHz on GaAs substrate 0.127mm thick with ε_r = 12.9. The port width is 0.053λ. (15 ports along each edge). .................................................. 37

3.6 Computed Z_{11} for a 50Ω microstrip line on GaAs substrate for different numbers of interconnected ports ......................................................... 38

3.7 Radiated power as a function of the length of the microstrip line sections on each side of a right-angled bend ................................................. 39

3.8 Voltage distribution at 15 GHz along the edges of an asymmetric step discontinuity......................................................... 66

3.9 Voltage distribution at 16 GHz along the edges of an asymmetric step discontinuity ......................................................... 67

3.10 Voltage distribution at 17 Ghz along the edges of an asymmetric step discontinuity ......................................................... 68

3.11 S_{11} results for a 20Ω open end on ε_r = 2.2 substrate, 10 mil thick, using P-Mesh Code......................................................... 70
3.12 S-parameters for a right-angled bend obtained by using the P-Mesh Code. .......................................................... 71

3.13 Power radiated results for a 50Ω right-angled bend. ................. 71

4.1 Comparison of measured and calculated resonant frequency. ............... 77

4.2 Calculated resonance frequency radiated power and Q-factors for the resonant structures. .................................................. 79

4.3 Measured results for microstrip resonators. ............................. 81

4.4 Measured loaded and unloaded Q-factors ................................. 81

4.5 Comparison of measured and calculated radiation Q-factors. ...... 82

5.1 Comparison of coupling $S_{13}$ (in dB) computed by MNM approach with quasistatic results for two parallel strips. ................. 99

6.1 Comparison between calculated and measured results at 10 GHz of coupling between two parallel lines with step discontinuity. .............. 115

7.1 Coupling results $S_{14}$, between two corners of right-angled bends for different value of the distribution width. ....................... 118

8.1 C-matrix for the lumped network modeling the gap. .................. 139

8.2 L-matrix for the lumped network modeling the gap. .................. 143

8.3 L-network for modeling H-field coupling. ............................... 146

8.4 Cut-off frequency in (GHz) for higher-order modes in coupled lines. ............................................................................. 155

9.1 S-parameters for a coupled line section with two right-angled bends as a function of $X_0$ for 2.5mm spacing between the strips. 191

9.2 S-parameters for a coupled line section with two right-angled bends as a function of $X_0$ for 3mm spacing between the strips. 191

9.3 Differences in $S_{12}$ between curves A, B, D and curve C. .......... 194

9.4 Differences in spurious coupling $S_{12}$ between curves B, C and curve A. ................................................................. 199

10.1 $S_{12}$ results between two open ends in an enclosure, as shown in Figure 10.5, for different numbers of modes. .................. 211
10.2 $S_{13}$ results between two right-angled bends in an enclosure, as shown in Figure 10.8, for different numbers of modes. .......... 211

C.1 Spectral domain numerical algorithm. ......................... 246

F.1 $S$-parameter results in dB for an ideal coupler with $(s/h = 0.5)$ for different numbers of ports in the gap region. .... 273

F.2 $S$-parameter results in dB for an ideal coupler with $(s/h = 1)$ for different numbers of ports in the gap region. .............. 273

F.3 $S$-parameter results in dB for an ideal coupler with $(s/h = 15.75)$ for different numbers of ports in the gap region. .. 273
CHAPTER I

INTRODUCTION

1.1 Background

Microstrip circuits and antennas possess attractive features such as low profile, light weight and small volume and low production cost. During recent years, great progress has been made in the design and development of microwave and millimeter wave microstrip circuits for use in communications links, guiding systems and seekers.

Higher operating frequencies demand more precision in the design of microstrip circuits and antennas. This is very important for monolithic microwave integrated circuits (MMICs) in which case the fabrication process is time consuming and costly.

In monolithic microwave integrated circuits (MMICs), transmission line discontinuities and junctions always exist. Any deviation from straight uniform microstrip lines, such as T-junctions, bends and steps in width, causes the introduction of discontinuity effects which must also be modeled as accurately as possible. The effect of discontinuities become more critical at higher frequencies or when circuit density is increased and discontinuities are located close to each other. A complete understanding of the design of microstrip circuits and microstrip antenna arrays requires characterization of the discontinuities'
effects. Since it is impossible to make adjustments in GaAs monolithic integrated circuits after they have been fabricated, an accurate and comprehensive modeling of each device and circuit element is required to save expensive and time consuming iteration of mask and wafer fabrication and evaluation. The major effects of discontinuities on circuit performance are: additional signal loss in the circuit due to mismatch losses and due to radiation losses and undesired interaction between different parts of the circuit due to external electromagnetic coupling. Reactive effects caused by discontinuities (and resulting in mismatch) has been studied extensively [1-4].

Because of the open nature of the microstrip configuration, hybrid and monolithic microwave circuits suffer from radiation originating at various geometrical discontinuities. Two consequences of this radiation phenomenon are: (i) additional signal loss in the circuit, and (ii) undesired interactions between different parts of the circuit due to external electromagnetic coupling. These phenomena become significant in two different situations. Firstly, when attempts are made to increase circuit density in monolithic microwave circuits, more bends and other discontinuities are introduced and spurious electromagnetic coupling increases considerably. Secondly, in microstrip antenna arrays, relatively thicker substrates are used and the feed network printed on the same substrate can result in substantial spurious radiation.

The purpose of this thesis is to investigate radiation loss and spurious coupling among microstrip discontinuities of generalized shapes.

1.2 Previous Work on Radiation and Spurious Coupling Caused by Discontinuities

Estimates of the radiation loss from microstrip discontinuities have been attempted previously [5-11]. Most of these results are based on the Poynting
vector method developed by Lewin [5]. In this approach, a line current located at the middle of the microstrip line is taken as a source of radiation. Thus the method is applicable primarily to narrow microstrip lines. It has been applied to a 90° bend, a step discontinuity, an open-end, a line terminated in an impedance, and a matched symmetrical T-junction. More accurate estimates based also on the Poynting vector method, of the power radiated from open circuit and short circuit microstrip discontinuities and from a mismatched termination were reported in [8-9]. In [11] the Poynting vector method is used to derive closed form expressions, using the computer algebra solver (MACSYMA), from microstrip bends and Y-junctions. The approach is not easily extendable to more complicated geometries found in microwave circuits.

In the recent years the need for more accurate models for characterization of microstrip discontinuities has become essential in view of the current advances in and popularity of GaAs MMIC technology. In existing computer-aided design packages for designing microstrip circuits, discontinuities are modeled by lumped element equivalent circuits. Microstrip discontinuities such as bends, open-ends, steps in width, gaps and T-junctions may be represented by an equivalent lumped circuits as described in [12,13]. A discontinuity in microstrip is caused by abrupt change in the geometry of the strip conductor. Therefore electric and magnetic field distributions are modified near the discontinuity. The altered electric field distribution give rise to a change in the equivalent capacitance, and the change in the magnetic field distribution can be expressed in terms of an equivalent inductance. The analysis of microstrip discontinuities involves the evaluation of these capacitances and inductances. Analysis of microstrip discontinuities can be based on quasi-static considerations [14-16] or carried out more rigorously by full wave analysis [17]. However,
to yield accurate results, it is not only crucial to have accurate reactive models for the different junctions and discontinuities, but effects such as radiation loss and spurious coupling due to radiation must also be accounted for in the simulation process.

Full wave analysis has been used [18-20] to analyze microstrip discontinuities. In the full wave analysis approach a mixed potential integral equation is solved to characterize the fields everywhere for a microstrip structure. This approach is the most rigorous and comprehensive approach because effects such as radiation and mutual coupling are included implicitly by using the exact Green's functions for microstrip structures. The unknown in the integral equation is the two-dimensional surface current distribution on the microstrip configuration. The solution for the current is found by first enforcing the boundary conditions and then applying the method of moments to the integral equation. The integral equation is reduced to a matrix equation and the unknown current values are obtained using LU decomposition and back substitutions. The effect of the junction is parameterized by probing the current away from the junction. This method has been applied to analyze various configurations such as double-bend circuits, double stub circuits, interdigitated capacitor and gap discontinuity.

Sections of coupled microstrip lines are used extensively for design of directional couplers, filters and other components in hybrid and monolithic microwave circuits [21,22]. Although characteristics of uniform coupled lines have been studied extensively [23], characterization of discontinuities and junctions in coupled lines is not readily available. Open-end is perhaps the only coupled microstrip line discontinuity analyzed in detail [24]. Specifically, there is need for the characterization of junctions between the coupled line sections and the
outgoing single microstrip lines connected to other components in the circuit.

Results for radiated power from a compensated bend, gap discontinuity and coupled microstrip line discontinuities are not available in literature; nor are results for spurious coupling among coupled microstrip line discontinuities. The Multiport Network Model (MNM) and the Planar-Lumped Model (PLM) described in this thesis are also employed to evaluate radiation from these discontinuities and to evaluate spurious coupling among coupled microstrip line discontinuities.

1.3 Approach Used in the Present Investigation

In this thesis a planar multiport network model of the discontinuity configuration and the segmentation method are used to evaluate radiation loss and spurious coupling among microstrip discontinuities in microstrip circuits.

1.3.1 Two-Dimensional Planar Model

Various types of components for microwave circuits may be classified in three categories. Lumped elements are much smaller than the wavelength, corresponding to the operating frequency, and may be termed as zero-dimensional. Transmission line (coaxial lines stripline and microstrip line) components may be called one-dimensional since their cross-sectional dimensions are much smaller than the wavelength, whereas their lengths are comparable to wavelength. Dimensions of waveguide components are comparable to wavelength and may be called three-dimensional components. In another group of components the height is much smaller than the wavelength but the other two dimensions are comparable to wavelength. These components are called planar or two-dimensional components [25]. The concept of 2-d planar circuits has found several applications in microwave integrated circuits, reduced height waveguide
circuits and microstrip antennas. The major advantages of the planar circuit approach are as follows:

- One-dimensional stripline and microstrip line circuits may be treated as planar circuits. At high frequencies the line width becomes large for low impedance level and is comparable to wavelength. In such cases the planar circuit approach yields better characterization of the circuit than offered by the transmission line approach.

- The planar circuit approach is also applicable to the analysis of discontinuities in striplines and microstrip lines.

- In microstrip antennas, the aperture field distribution can be evaluated by treating them as planar components with magnetic walls. In most applications, the thickness of the substrate used for microstrip antennas and circuits is small compared to the operating wavelength. Therefore the fields can be assumed to be constant along the height of the substrate. At the strip conductor and at the ground plane the tangential components of the electric field are zero. A magnetic wall is assumed to exist at the periphery of the planar component. Since the fields do not vary along the height of the substrate the transverse electric field components equal zero within the dielectric region. The fringing field at the edges may be taken into account by shifting the magnetic wall by a certain distance from the physical edges or by incorporating equivalent edge admittance networks.
1.3.2 Multiport Network Model

In the multiport network approach for microstrip antennas, fields underneath the microstrip configuration, the external fields (radiated fields, surface waves) are modeled separately in terms of multiports subnetworks which are characterized in terms of Z-matrices or Y-matrices. These subnetworks are combined using the segmentation network analysis techniques to obtain the circuit characteristics such as scattering parameters. In order to implement this model to investigate radiation loss and spurious coupling among microstrip discontinuities, we add a number of open ports at the edges of the discontinuity structure. Green's function approach and the segmentation method are used to evaluate the multiport Z-matrix for the discontinuity. This multiport Z-matrix is used to calculate voltages at the N edge ports for a unit current input at the input port. This voltage distribution is expressed as an equivalent magnetic current line source distribution which is used to calculate the far zone field for radiation loss or the near field at the location of the other discontinuity for spurious coupling calculations. The spurious coupling between two discontinuities (due to external fields) may be incorporated in the analysis by connecting an additional multiport network between the two discontinuities.

To characterize coupled microstrip line discontinuities a Planar Lumped Model for coupled microstrip sections has been developed in this research. Fields underneath the two strips and those fringing at the outer edges are modeled by equivalent planar waveguides. Electric and magnetic field coupling in the gap region is modeled by a lumped network.
1.3.3 Segmentation Method

The segmentation technique was first proposed in [26] and was implemented to analyze microwave circuits using the S-matrix formulation. Later it has been shown [27] that a Z-matrix formulation is more suitable for planar circuits, since the Z-matrix follows directly from the Green's function approach. For the analysis of microstrip discontinuities reported in this thesis, the Z-matrix formulation is used for all segments. The application of segmentation formulation to combine two segments is described in detail in Appendix B.

1.4 Organization of the Thesis

In Chapter Two, a numerical technique for evaluating radiation loss from microstrip discontinuities is presented.

In Chapter Three, numerical results of voltage distribution along the edges of various microstrip discontinuities are discussed. Results of radiated power from various discontinuities are also given. These results are in good agreement with the Poynting Vector Method [5-7]. MNM results for power radiated from an open end are compared to fullwave analysis results in Section 3.12.

In Chapter Four, an experiment to verify the radiation from a bend discontinuity is described. A comparison between computed and measured results is given.

In Chapter Five the multiport network model for evaluating spurious coupling between microstrip discontinuities is presented. The spurious coupling between two discontinuities (due to external fields) may be incorporated in the multiport network model by connecting an equivalent coupling network between the discontinuities. Results for spurious coupling have been verified
by considering the case of two quarter-wave parallel strips and comparing the numerical results by the proposed method with those computed on the basis of quasistatic analysis as described in Section 5.4.

Experimental verification of the proposed mutual coupling model is described in Chapter Six. Two different experimental verifications of the procedure are described in this chapter. Coupling among two lines (each incorporating two step discontinuities) was measured experimentally and compared with results obtained using MNM. The second experimental verification is obtained by comparing the results obtained using MNM with the measured results for a double-stub circuit obtained from Texas Instruments.

Numerical results for coupling between two right-angled bends, between two corners of right-angled bends and between two open-ends are given in Chapter Seven. Spurious coupling results among three discontinuities are also described in this chapter.

In Chapter Eight the planar-lumped model for coupled microstrip line discontinuities is described and applied to investigate single section couplers and a three-section coupler.

In Chapter Nine the planar-lumped model is used to evaluate radiation loss from coupled microstrip line discontinuities and to calculate spurious coupling caused by coupled microstrip line discontinuities.

The effect of an enclosure on parasitic coupling between microstrip discontinuities is described in Chapter Ten. This analysis makes use of the modal expansion of fields in cavities and the multiport network model. Results for coupling between two open ends in an enclosure are compared to fullwave analysis results in Section 10.4.
CHAPTER II

RADIATION LOSS FROM MICROSTRIP DISCONTINUITIES

2.1 The Approach

This chapter presents a method for estimating radiation from microstrip discontinuities of generalized shapes. A planar multiport network model of the discontinuity configuration and the segmentation method are used to evaluate voltage distribution around the edges of the discontinuity. This voltage distribution is expressed as an equivalent magnetic current line source distribution which is used to calculate the far-zone field (for radiation loss). Numerical results have been obtained for radiation from bends, steps, T-junctions, open-end stubs and gap discontinuities and are found to be in reasonable agreement with the Poynting vector results wherever available.

2.2 Planar Model for Microstrip Discontinuities

Multiport modeling of microstrip discontinuity configurations [3] is based on parallel plate waveguide model [28] for microstrip lines. A similar network modeling approach has been used earlier for analysis of microstrip patch antennas [29].

The planar waveguide model consists of two parallel conductors bounded by magnetic walls in the transverse directions. The electric and magnetic fields inside the planar model are uniform along the height of the substrate as shown
Figure 2.1: Planar waveguide model for microstrip line.
in Figure 2.1. The width of the waveguide model \( W_e(f) \) is made larger than the physical microstrip width in order to account for the fringing fields at the edges and is given by

\[
W_e(f) = \frac{\eta_0 h}{Z_0(f)\sqrt{\varepsilon_{re}(f)}}
\]  

(2.1)

where \( Z_0(f) \) is the characteristic impedance, \( \varepsilon_{re}(f) \) is the effective dielectric constant, \( h \) is the substrate height and \( \eta_0 \) is the wave impedance in free space.

As the frequency dependences of \( Z_0 \) and \( \varepsilon_{re} \) are incorporated in (2.1), the dispersion effects get built in the planar model also. The planar models for discontinuity configuration are obtained by replacing the physical widths of the microstrip lines by effective widths given by (2.1). The effective dimensions in the discontinuity region are obtained by extrapolating the effective edges for the connecting lines as shown for a 90° bend in Figure 2.2(a) or having an outward extension equal to that for the adjoining microstrip line (as shown in Figure 2.2(b). It has been shown that the planar model of a microstrip line is valid for higher order modes also [28]. Since discontinuities reactances are caused by excitation of higher order modes, the planar model holds good for characterization of discontinuities also. The planar modeling approach has been used extensively for characterization and compensation of microstrip discontinuities [1-3].

![Figure 2.2: Planar model for microstrip bends.](image)
2.3 Multiport Model for Microstrip Discontinuities

In this section, the multiport network approach for the analysis of microstrip circuits is discussed. In this model, the fields underneath the microstrip configuration and external fields (radiated, surface wave and fringing fields) are modeled separately in terms of multiport subnetworks which are characterized in terms of $Z$-matrices or $Y$-matrices. These subnetworks are then combined using segmentation technique to obtain the circuit scattering parameters. In this thesis the time dependence $e^{j\omega t}$ is assumed for the fields. The substrate material is assumed to be non-magnetic. The ground plane and the dielectric substrate are assumed to be infinite in extent.

2.3.1 Fields Underneath the Microstrip Circuit

In most applications, the thickness of the substrate used for microstrip circuits is small compared to the operating wavelength. Fields at the edges of the microstrip line decay rapidly away from the microstrip line edges. So, a solution of the electromagnetic fields in the region between the strip and the ground plane can be obtained by considering the microstrip line as a two-dimensional cavity with magnetic wall boundaries. The fringing fields near the edges are accounted for by extending the physical microstrip width as described in Section 2.2.

For microstrip configurations of regular shapes (rectangles, circles, rings, sectors of circles, sectors of rings and triangles), the multiport planar component representing the microstrip configuration (with open-circuited ports on the periphery) can be analyzed by using two-dimensional impedance Green's functions available for these shapes as discussed in Appendix A.

The effect of the dielectric losses is incorporated in the analysis by con-
sidering a complex dielectric constant. Conductor losses are included in the analysis by defining an equivalent loss tangent $\delta_c$ as [21]

$$\delta_c = \sqrt{2/\omega\mu_0\sigma/h}$$ (2.2)

where $\sigma$ is the conductivity of the strip, $h$ is the substrate height and $\mu$ is the permeability in free space.

2.3.2 External Fields

In the MNM for microstrip circuit external fields (radiation fields and surface wave fields) may be incorporated in the model by adding an equivalent edge admittance network (EAN) connected to the edges of the microstrip circuit. The concept of representing the radiated fields by an edge admittance has been used in [30, 31]. The edge conductance $G$ in an edge admittance network consists of a radiation conductance $G_r$ and a surface wave conductance $G_s$. The radiation conductance associated with an edge of a microstrip circuit is defined as an ohmic conductance, which when connected to the edge (continuously or at discrete ports) will dissipate a power equal to that radiated by the microstrip configuration. For a uniform voltage distribution along an edge, the radiation conductance may be written as

$$G_r = \frac{2P_r}{V^2}$$ (2.3)

where $P_r$ is the radiated power from the edge and $V$ is the voltage along the edge. For a non-uniform voltage distribution $V(s)$ along the edge, the radiation conductance may be written as

$$G_r = \frac{P_r}{\frac{1}{W} \int_0^W V^2(s) ds}$$ (2.4)
where \( s \) denotes the distance along the edge of the circuit and \( W \) is the port width. In microstrip circuit bends and other discontinuities are introduced and the voltage distribution along the edges cannot be represented by a continuous function but can be calculated at discrete points. In this case the integral in equation (2.4) may be represented as a summation and the radiation conductance may be written as

\[
G_r(i, i) = \frac{1}{n} \sum_{i=1}^{n} \frac{2P_r}{V_i^2}
\]

(2.5)

where \( V_i \) is the edge voltage at port \( i \) and \( n \) is the number of ports along the edge.

From equation (2.5) elements of the edge impedance matrix for a microstrip circuit are obtained as:

\[
Z_r(i, i) = n \sum_{i=1}^{n} \frac{V_i^2}{2P_r}
\]

(2.6)

The edge impedance matrix is a diagonal matrix representing radiation loss.

The surface wave conductance \( G_s \) is defined in a similar manner and may be written as

\[
G_s = \frac{2P_{sur}}{\frac{W}{V} \int_0^W V^2(s) \, ds}
\]

(2.7)

where \( P_{sur} \) is the power coupled to the surface waves excited by the voltage distribution \( V(s) \) at the edge.

For a discrete voltage distribution elements of the surface wave impedance matrix may be written as

\[
Z_{sur}(i, i) = n \sum_{i=1}^{n} \frac{V_i^2}{2P_{sur}}
\]

(2.8)

For thin dielectric substrates \( G_s \) is much smaller than \( G_r \) and may be neglected [32].
2.3.3 MNM for Evaluating Edge Voltages

In order to evaluate the external fields produced by a microstrip discontinuity, we first obtain voltages at the edges of the microstrip discontinuity structure. A multiport network model, similar to that developed for microstrip patch antennas [29-30] is employed. For implementing this method, we add a number of open ports at the edges of the discontinuity structure from which the radiation (or spurious external coupling) is being evaluated.

This is shown in Figure 2.3 for a right-angled bend and a compensated right-angled bend.

Figure 2.3: Multiple ports located at the edges of the microstrip discontinuities.

Lengths of transmission lines on two sides of the junction are taken large enough so that the higher order evanescent modes produced by the discontinuities decay out at the locations of external ports 1 and 2 shown in Figure 2.3. The circuit behavior is simulated by terminating the port 2 in a matched load and adding a matched source to the port 1. Voltages at the $N$ ports at the edges are computed by using the following procedure:
(i) The configuration is broken down into elementary regular segments, connected together at the interfaces by a discrete number of interconnections.

(ii) $Z$-matrices for each of these elementary segments are evaluated by using the Green's function approach for individual geometries. This approach is summarized in Appendix A.

(iii) Individual $Z$-matrices obtained in (ii) are combined together by using the segmentation formula, given in Appendix B.

(iv) Overall multiport $Z$-matrix is used for calculating voltages at the $N$ edge ports for a unit current input at the input port.

As mentioned earlier, a similar procedure has been used for design of microstrip patch antennas [30]. The only distinction in the latter case is the use of edge admittance networks (containing equivalent radiation conductances) which are connected to the edge ports. Because of non-resonant nature of the microstrip discontinuity structures, the radiated power is small and the edge voltages may be assumed to be unaffected by radiation conductances involved. However, when the radiation power is greater than -10 dB (relative to the input power), the radiation conductance networks are added to the ports along the edges and iterative computations are carried out for evaluating voltage distribution along the edges and radiation fields. For calculations of power radiated from resonators, two to three iterations are needed until convergence of edge voltages is achieved.

2.4 Radiation Fields of a Magnetic Current Element

Voltages at the discontinuity edges are represented by equivalent magnetic current sources as shown in Figure 2.4. Each of the magnetic current line
sources is divided into small sections over which the field may be assumed to be uniform. The amplitude \( M \) of each of the magnetic current elements is twice that of the edge voltage at that location (to account for image with respect to the perfectly conducting ground plane) and the phase of \( M \) is equal to the phase of the corresponding voltage.

![Figure 2.4: Equivalent magnetic current distribution at discontinuity edges.](image)

The fields produced by a magnetic current element \( M dl \) can be derived by using the concept of electric vector potential \( F \) which is the dual of magnetic vector potential \( A \). Electric vector potential \( F \) [33] is defined by

\[
\varepsilon E = -\nabla \times F \tag{2.9}
\]

\[
F = \frac{1}{4\pi} \int \frac{\varepsilon M e^{-jkr}}{r} dV \tag{2.10}
\]

where \( k \) is the wave number in free space. For a \( z \)-directed magnetic current element of length \( dl \) we have

\[
dF_i = \frac{\varepsilon M dl e^{-jkr}}{4\pi r} \tag{2.11}
\]
Figure 2.5: Fields produced by a magnetic current element.

From Figure 2.5

\[ F_r = F_z \cos \theta \]  \hspace{1cm} (2.11.a)

\[ F_\theta = -F_z \sin \theta \]  \hspace{1cm} (2.11.b)

\[ F_\phi = 0 \]  \hspace{1cm} (2.11.c)

Because of symmetry, \( \frac{\partial}{\partial \phi} = 0 \), so by applying (2.9) to (2.10) we obtain

\[ E_\phi = -\frac{M}{4\pi} \frac{d\ell}{r^2} \sin \theta \left( \frac{jk e^{-jk\ell}}{r} + \frac{e^{-jk\ell}}{r^2} \right) \]  \hspace{1cm} (2.12)

The magnetic field \( H \) can be obtained from Maxwell’s equations

\[ \nabla \times E = -j\omega \mu H \]  \hspace{1cm} (2.13)

Taking the curl of (2.12) we obtain

\[ H_\theta = \frac{M}{4\pi \eta} \frac{d\ell \sin \theta}{r} \left( \frac{jk}{r} + \frac{1}{r^2} + \frac{j}{kr^3} \right) e^{-jk\ell} \]  \hspace{1cm} (2.14)

\[ H_r = \frac{M}{2\pi \eta} \frac{d\ell \cos \theta}{r} \left( \frac{1}{r^2} + \frac{1}{jkr^3} \right) e^{-jk\ell} \]  \hspace{1cm} (2.15)

\[ \eta = \sqrt{\frac{\mu}{\varepsilon}} \]  \hspace{1cm} (2.16)
For the far zone radiation field we retain only the terms with \( \frac{1}{r} \) variation.

2.5 Radiation Fields of a Magnetic Line Source

From Figure 2.6,

\[
R = r - Z \cos \theta
\]  

(2.17)

Using equations 2.17 and 2.14 (with \( \frac{1}{r} \) variation only for \( H_\theta \)) we can write for a line source of length \( W \):

\[
H_\theta = \frac{j k M}{4 \pi \eta} \int_{-\frac{W}{2}}^{\frac{W}{2}} \frac{\sin \theta}{R} e^{-j k R} \, dz
\]  

(2.18)

![Figure 2.6: A magnetic line source and coordinate system for evaluating radiation fields.](image)

Substituting (2.17) in \( e^{-j k R} \), letting \( R \approx r \) in the denominator, and carrying out the integration we have

\[
H_\theta = \frac{j k M W}{4 \pi \eta r} F(\theta) \, e^{-j k r}
\]  

(2.19.a)

where \( F(\theta) \) is given by

\[
F(\theta) = \left( \sin \left( \frac{k_0 W}{2} \cos \theta \right) / \sin \theta \right) \sin \theta
\]  

(2.19.b)
2.6 Radiated Power from Microstrip Discontinuities

The total radiation is computed using the superposition of the far field radiated by each section. Referring to the coordinate system, shown in Figure 2.7, the far-field pattern \( F(\theta, \phi) \) may be written in terms of voltages at the various elements. With the voltage at the i-th element given as \( V(i)e^{j\omega t} \), we have

\[
F(\theta, \phi) = \sum_{i=1}^{N} 2V(i)W(i)\exp\{k_0\gamma_0(i) + \alpha(i)\}F_i(\theta, \phi)
\]

(2.20)

where

\[
F_i(\theta, \phi) = \frac{\sin\left(\frac{k_0W(i)}{2} \cos \theta\right)}{\frac{k_0W(i)}{2} \cos \theta} \sin \theta
\]

\[
\gamma_0(i) = X_0(i) \sin \theta \cos \phi + Y_0(i) \cos \theta
\]

\( N \) is the number of ports, \( X_0(i), Y_0(i) \) specify the location of the i-th magnetic current element, \( k_0 \) is the free space wavenumber and \( W(i) \) is the width of the i-th element. The factor of 2 in equation (2.20) accounts for the image of the magnetic current with respect to the ground plane.

The radiated power \( P_r \), calculated by the integration of the Poynting vector over the half space, may be written as:

\[
P_r = \frac{1}{240\pi} \int_{-\pi}^{\pi} \int_{0}^{\pi} (|E_\theta|^2 + |E_\phi|^2) r^2 \sin \theta d\theta d\phi
\]

(2.21)

The fields \( E_\theta \) and \( E_\phi \) are expressed in terms of \( F(\theta, \phi) \) as:

\[
E_\theta = \hat{\alpha}_\theta \left( -\frac{j k_0}{4\pi r} F(\theta, \phi) F_\theta \right)
\]

(2.22)

\[
E_\phi = \hat{\alpha}_\phi \left( -\frac{j k_0}{4\pi r} F(\theta, \phi) F_\phi \right)
\]

(2.23)

where

\[
F_\phi = \sin \phi' \sin \phi + \cos \delta \cos \phi \cos \phi'
\]

(2.24)

\[
F_\theta = -\sin \phi' \cos \theta \cos \phi + \cos \delta \cos \theta \sin \phi + \sin \delta \cos \phi' \sin \theta
\]

(2.25)
Figure 2.7: Coordinate system for external field calculations.

\[
\cos \theta' = \sin \theta \sin \phi \sin \delta + \cos \theta \cos \delta \tag{2.26}
\]

\[
\cos \phi' = \sin \theta \cos \phi / \sqrt{1 - \cos^2 \theta'} \tag{2.27}
\]

The radiation loss may be expressed as:

\[
\text{Radiation loss (dB)} = 10 \log_{10}(1 - \frac{P_r}{P_t}) \tag{2.28}
\]

where \( P_t \) is the input power at port 1. \( P_t \) is calculated from input current and the input impedance of the discontinuity configuration terminated in matched loads at other ports.
2.7 Incorporation of Radiation Loss in S-Parameters of Discontinuities

The radiation loss may be incorporated in the S-parameters representation of the discontinuities by combining the Z matrix of the multiport model of the discontinuity with the impedance matrix for the edge impedance network. Figure 2.8 shows a simple edge admittance network connected to a bend. The overall Z-matrix may then be converted to the corresponding S-matrix. Derivation of the edge impedance matrix for a discontinuity configuration is described in Section 2.3.

2.7.1 Sample Results for S-Matrix

The SWR results without radiation loss and with radiation loss for a 50Ω bend on 0.01 inch substrate with \( \epsilon_r = 2.2 \) at a frequency of 40 GHz, are given in Table 2.1. Elements of the S matrix for the bend with and without radiation loss are also included in this Table.

2.8 Approximations Involved in MNM Approach

The approximations involved in the multiport network model are discussed and justified in this section. The major approximations involved in MNM approach are as follows:

- Two-dimensional fields;
- Matching the fields at interconnecting ports;
- Surface waves are not accounted for in the model;
- Magnetic current modeling.

2.8.1 Two-Dimensional Fields

In most applications, the thickness of the dielectric substrate used for
Figure 2.8: Multiport network model for incorporating radiation loss in S-parameters.
Table 2.1: Effect of Radiation on SWR and S-parameters

<table>
<thead>
<tr>
<th>f(GHz)</th>
<th>SWR without Radiation Loss (dB)</th>
<th>SWR with Radiation Loss (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>-21.030</td>
<td>-21.025</td>
</tr>
<tr>
<td>20</td>
<td>-14.540</td>
<td>-14.557</td>
</tr>
<tr>
<td>30</td>
<td>-10.208</td>
<td>-10.252</td>
</tr>
<tr>
<td>40</td>
<td>-6.550</td>
<td>-6.630</td>
</tr>
</tbody>
</table>

S-Matrix Radiation Loss Not Included

\[ S = \begin{bmatrix} 0.47 \angle -174.58^\circ & 0.879 \angle -84.61^\circ \\ 0.879 \angle -84.61^\circ & 0.47 \angle -174.58^\circ \end{bmatrix} \]

S-matrix with Radiation Loss Included

\[ S = \begin{bmatrix} 0.465 \angle -175.51^\circ & 0.87 \angle -84.32^\circ \\ 0.87 \angle -84.32^\circ & 0.465 \angle -175.51^\circ \end{bmatrix} \]

Microstrip circuits is small compared to the operating wavelength. Therefore, the fields can be assumed to be constant along the height of the substrate.

The general Helmholtz equation (three-dimensional), valid for a source-free region, is given as

\[(\nabla^2 + k^2)E = 0\]  \hspace{1cm} (2.29)

where \(k^2 = \omega^2 \mu \varepsilon\) where \(\mu\) and \(\varepsilon\) are the permeability and permittivity of the dielectric material and \(\omega\) is the angular frequency. Referring to Figure 2.1(a) for thin dielectric substrates \(\frac{\partial}{\partial z}\) is equal to zero and equation (2.29) may be written as

\[(\nabla_f^2 + k^2)E = 0\]  \hspace{1cm} (2.30)

where \(\nabla_f^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\). At the center conductor and at the groundplane the
tangential components of the electric field are zero. Magnetic walls may be placed along the periphery of the microstrip circuit, since the fields do not vary along the height of the substrate. The transverse fields \((E_x, E_y)\) equal to zero within the dielectric region. The fringing field at the edges is taken into account by shifting the magnetic wall, as described in Section 2.2 and as shown in Figure 2.1, by a certain distance from the physical edges.

Since \(E_x\) and \(E_y\) are zero, the E-field in two-dimensional analysis of microstrip circuits may be written as

\[ E = \hat{a}_x, E_z(x,y) \]  

(2.31)

where \(\hat{a}_x\) is a unit vector along the \(z\)-direction.

The characterization of the planar components can be carried out in terms of an RF voltage \(V\) on the central conductor. Since \(\partial E_z/\partial z\) equal zero using (2.31) the voltage \(V\) may be written as \(V = -E_z h\). Equation (2.30) may be written as

\[(\nabla^2 + k^2)V = 0 \]  

(2.32)

with \[ \frac{\partial V}{\partial n} = 0 \]

where \(n\) is a unit vector normal to the boundary.

### 2.8.2 Matching the Fields at Interconnecting Ports

Matching the fields at the discontinuity edges and at the interconnecting ports is achieved by satisfying Kirchhoff's laws at these interconnecting ports. Equating the voltages at the connected ports is equivalent to matching the tangential electric field, and the continuity of currents ensures the continuity of the magnetic field at the interconnecting ports.
These interconnection conditions are applied at a discrete number of interconnected ports. The number of interconnected ports is determined by iterative computations by increasing the number of interconnected ports until the calculated voltage distribution along the edges converge (i.e. do not change with further increase of the number of ports).

2.8.3 Surface Waves Are Not Accounted for in the Model

In the edge impedance matrix introduced in Section 2.3.2 surface wave effects may be included as described in Section 2.3.2. As discussed in [32] for thin dielectric substrates $G_s$ is much smaller than $G_r$ and may be neglected. In all the examples described in this thesis the height of the substrate is less than 0.05 times the wavelength. Hence, surface wave effects may be neglected.

2.8.4 Magnetic Current Modeling

Magnetic current density $M$ is a fictitious quantity defined to make use of consequent duality in Maxwell's equations. Maxwell's equations may be written as [33]

$$\nabla \times H = \frac{\partial D}{\partial t} + J \quad (2.33)$$

$$\nabla \cdot D = \rho_e \quad (2.34)$$

$$\nabla \times E = -\frac{\partial B}{\partial t} - M \quad (2.35)$$

$$\nabla \cdot B = \rho_m \quad (2.36)$$

where the magnetic current density $M$ and the magnetic current charge density $\rho_m$ are fictitious quantities introduced artificially to make equations (2.35) and (2.36) similar to (2.33) and (2.34) respectively. The boundary condition for the tangential magnetic field may be written as

$$J_s = \hat{n} \times (H_1 - H_2) \quad (2.37)$$
where \( \hat{n} \) is a unit vector normal to the surface. When region 2 is a perfect conductor, \( H_2 \) is equal to zero.

\[
J_s = \hat{n} \times H_1
\]  

(2.38)

In a similar manner if a linear magnetic current density \( M_s \) is located at an interface between two media, 1 and 2, the tangential components of the electric field will be discontinuous by an amount given by

\[
M_s = -\hat{n} \times (E_1 - E_2)
\]  

(2.39)

If the medium 2 is a perfect conductor \( E_2 \) equal to zero, and

\[
M_s = -\hat{n} \times E_1
\]  

(2.40)

The multiport network modeling approach is based on considering the upper surface of the microstrip circuit as a reference plane and evaluating the external fields in terms of equivalent sources on this reference plane. According to the Schelkunoff's equivalence theorem, we can postulate a conducting sheet located on this reference plane (considered a closed surface enclosing all of the sources) and locate the equivalent magnetic current sources on this conducting plane. The equivalent magnetic current sources are directly related to the tangential component of electric field at this reference plane. For a microstrip line configuration, this tangential electric field is normal to the microstrip edge and decays rapidly as we move laterally away from this microstrip edge. The equivalent current \( M \) is given by equation (2.40) and is directed parallel to the edge. The magnitude of this magnetic current line source is equal to the integral of the tangential E-field evaluated from the microstrip edge to an infinite distance. This E-field integral can be recognized as voltage at the microstrip edge. Thus,
equivalent magnetic current distribution may be evaluated from the distribution of voltages at the edges of the microstrip circuit. Referring to Figure 2.9(b) both $M_s$ and $J_s$ may exist over regions 1-2 and 3-4, while only electric current density $J_e$ is possible (since $E_z = 0$) over the conductor in region 2-3. In the implementation of Schelkunoff’s theorem used here (medium enclosed by surface $S$ being replaced by a perfect conductor), only equivalent magnetic currents are needed for evaluating external fields. The magnitude of the magnetic current density $M_s$ is proportional to $E_z$ over $S$. For small values of $h$, $E_z$ decays rapidly as we move away from edges 2 and 3. Assuming $E_z$ to be $Z$-directed at the edges 2 and 3, the equivalent magnetic current at the edges are directed in $y$ and -$y$ direction as shown in Figure 2.9(c). For a uniform microstrip line radiation fields of these two magnetic current line sources cancel each other.

At the microstrip edges, voltage and hence the equivalent magnetic current may be written as

$$M = \int_0^h E \cdot dz = V \tag{2.41}$$

where $V$ is the voltage between the conductors. Since the substrate height $h$ is very small compared to the wavelength $E_z$ may be assumed constant over the distance $h$. Further small values of $h$ allow us to approximate distributions of magnetic currents by two equivalent line sources of magnetic currents as shown in Figure 2.9(d).
Figure 2.9: Magnetic current modeling of edge fields.
CHAPTER III

NUMERICAL RESULTS FOR RADIATION LOSS
FROM MICROSTRIP DISCONTINUITIES

3.1 Introduction

The multiport network model discussed in Chapter 2 has been applied to evaluate radiation loss from several microstrip discontinuities including: right-angled bends, step junctions, T-junctions, chamfered bends, 20Ω stub in 50Ω microstrip line, gap discontinuities and open ends. Some typical results are reported in this chapter.

The multiport network method results for radiation loss from microstrip discontinuities have been compared with the Poynting vector method (applicable for simple geometries) and also with fullwave analysis results in several cases.

3.2 Computational Details

In order to improve computation accuracy and to minimize computation time several computations were performed to determine the minimum number of terms in the Green's function expansion, the width of external and interconnected ports, and the extent of radiating regions for a discontinuity configuration.
3.2.1 Number of Terms in Green's Function Expansion

For rectangular planar segments and for right-angle isosceles triangular segments, the impedance matrix elements can be expressed in a single infinite series [34, 35]. The number of terms in Green's function expansion was found by computing the impedance matrix elements for a given configuration for different number of terms in the Green's function expansion and checking the convergence of the impedance matrix elements as a function of number of terms in the summation.

In [36] the input impedance at 3 GHz of a rectangular segment, with dimensions \((\frac{3\lambda}{8} \times \frac{3\lambda}{8})\) on a substrate with \(\varepsilon_r = 2.53\) and \(0.7\text{mm}\) thick, is computed using a double infinite series and a single infinite series. A summary of the results given in [36] is listed in Table 3.1. In the table the percentage error in the input impedance versus the computation time in cpu seconds is given. One may conclude that 100 terms for the single summation will assure good accuracy.

Table 3.1: Comparison of double and single infinite series for computation of elements of the \(Z\)-matrix for a rectangular element.

<table>
<thead>
<tr>
<th>Number of Terms</th>
<th>Double Summation Error (in %)</th>
<th>CPU Time (Seconds)</th>
<th>Single Summation Error (in %)</th>
<th>CPU Time (Seconds)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>25.0</td>
<td>0.2</td>
<td>8.0</td>
<td>0.08</td>
</tr>
<tr>
<td>10</td>
<td>13.0</td>
<td>0.7</td>
<td>1.0</td>
<td>0.2</td>
</tr>
<tr>
<td>20</td>
<td>6.0</td>
<td>5.0</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>30</td>
<td>4.0</td>
<td>6.0</td>
<td>0.4</td>
<td>0.6</td>
</tr>
<tr>
<td>60</td>
<td>3.0</td>
<td>10.0</td>
<td>0.2</td>
<td>0.8</td>
</tr>
<tr>
<td>100</td>
<td>2.0</td>
<td>60.0</td>
<td>0.0</td>
<td>3.0</td>
</tr>
<tr>
<td>150</td>
<td>1.5</td>
<td>70.0</td>
<td>0.0</td>
<td>5.0</td>
</tr>
</tbody>
</table>
A comparison between the single infinite series and double infinite series for a right angle isosceles triangular segment is given in [35]. The impedance matrix elements were computed at 60 GHz for a nine-port isosceles triangular segment on GaAs substrate with $\varepsilon_r = 12.9$ and $125\mu m$ thick. A summary of the results given in [35] is listed in Table 3.2. In this table the percentage error in $Z_{11}$ versus the computation time in cpu seconds is given. Also for a right angle isosceles triangular segment 100 terms for the single summation will assure good accuracy in computation of elements of the $Z$-matrix.

Table 3.2: Comparison of double and single infinite series for computation of the elements of the $Z$-matrix for an isosceles triangular segment.

<table>
<thead>
<tr>
<th>Number of Terms</th>
<th>Double Summation Error (in %)</th>
<th>CPU Time (Seconds)</th>
<th>Single Summation Error (in %)</th>
<th>CPU Time (Seconds)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>-</td>
<td>-</td>
<td>0.50</td>
<td>1</td>
</tr>
<tr>
<td>10</td>
<td>-</td>
<td>-</td>
<td>0.18</td>
<td>3</td>
</tr>
<tr>
<td>20</td>
<td>-</td>
<td>-</td>
<td>0.05</td>
<td>5</td>
</tr>
<tr>
<td>40</td>
<td>-</td>
<td>-</td>
<td>0.03</td>
<td>6</td>
</tr>
<tr>
<td>80</td>
<td>1.00</td>
<td>1000</td>
<td>0.01</td>
<td>8</td>
</tr>
<tr>
<td>100</td>
<td>0.80</td>
<td>2000</td>
<td>0</td>
<td>10</td>
</tr>
<tr>
<td>150</td>
<td>0.58</td>
<td>3000</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>200</td>
<td>0.40</td>
<td>6000</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

### 3.2.2 Port Width

As described in Section 2.3.3 the multiport network model is employed to evaluate the voltage distribution along the edges of the microstrip discontinuity by adding a number of open ports at the edges of the discontinuity structure. The width of these ports, and the number of ports taken, determines how accurately we can evaluate the variation in the voltage distribution along
the edges of the discontinuity configuration. If the port width is taken small enough we can assume that along the port width the voltage distribution is uniform.

To determine the port width needed to evaluate accurately the voltage distribution along the edges of the discontinuity we computed the voltage distribution at 10 GHz along the edges of a 50Ω microstrip line on GaAs substrate (with $\varepsilon_r = 12.9$, 127.1 µm thick) for different number of ports along the microstrip line edges. The length of the microstrip line was taken as 0.8 wavelength. Edge voltages were calculated for 10, 14 and 15 ports along the microstrip line length and are given in Tables 3.3 to 3.5. One may note that voltages at opposite ports along each edge are symmetric. A comparison between Table 3.3 and Table 3.4 shows that the differences in the amplitude of the edge voltages for port width of 0.08λ and for port width of 0.057λ are around 0.25%. A comparison between Table 3.3 and Table 3.5 shows that the differences in the amplitude of the edge voltages for port width of 0.08λ and for port width of 0.053λ are around 0.5%. One may conclude that the port width may be chosen as 0.08λ, since a narrower port width will not improve considerably the computed edge voltage distribution along the discontinuity configuration. In calculation reported in this thesis, the port width was chosen as 0.075λ or less.

As described in Section 2.8.2 matching the fields at the interconnecting ports is achieved by satisfying Kirchoff’s law at these interconnecting ports. These boundary conditions are applied at a discrete number of interconnected ports. The number of interconnected ports (or the interconnected port width) is determined by iterative computations by increasing the number of interconnected ports until the calculated voltage distribution along the edges converge. The optimum number of interconnected ports was found by computing $Z_{11}$ at
Table 3.3: Edge voltages for a 50Ω microstrip line 0.8λ long at 10 GHz on GaAs substrate, 0.127 mm thick with εr = 12.9, for port width of 0.08λ (10 ports along each edge)

**ELEMENTS OF MATRIX ZC**

**ELEMENTS OF COLUMN # 1**

<table>
<thead>
<tr>
<th>ROW#</th>
<th>MAGNITUDE</th>
<th>DEG.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.5010519409E+02</td>
<td>0.37</td>
</tr>
<tr>
<td>2</td>
<td>0.4951382446E+02</td>
<td>-59.39</td>
</tr>
<tr>
<td>3</td>
<td>0.4946280289E+02</td>
<td>-79.90</td>
</tr>
<tr>
<td>4</td>
<td>0.4944910049E+02</td>
<td>-100.39</td>
</tr>
<tr>
<td>5</td>
<td>0.4944556427E+02</td>
<td>-120.82</td>
</tr>
<tr>
<td>6</td>
<td>0.4942468262E+02</td>
<td>-141.22</td>
</tr>
<tr>
<td>7</td>
<td>0.4936354823E+02</td>
<td>-161.59</td>
</tr>
<tr>
<td>8</td>
<td>0.4926280975E+02</td>
<td>178.02</td>
</tr>
<tr>
<td>9</td>
<td>0.4913779068E+02</td>
<td>157.59</td>
</tr>
<tr>
<td>10</td>
<td>0.4902156685E+02</td>
<td>137.11</td>
</tr>
<tr>
<td>11</td>
<td>0.4893605804E+02</td>
<td>116.60</td>
</tr>
<tr>
<td>12</td>
<td>0.4951382446E+02</td>
<td>-59.39</td>
</tr>
<tr>
<td>13</td>
<td>0.4946280289E+02</td>
<td>-79.90</td>
</tr>
<tr>
<td>14</td>
<td>0.4944910049E+02</td>
<td>-100.39</td>
</tr>
<tr>
<td>15</td>
<td>0.4944556427E+02</td>
<td>-120.82</td>
</tr>
<tr>
<td>16</td>
<td>0.4942468262E+02</td>
<td>-141.22</td>
</tr>
<tr>
<td>17</td>
<td>0.4936354823E+02</td>
<td>-161.59</td>
</tr>
<tr>
<td>18</td>
<td>0.4926280975E+02</td>
<td>178.02</td>
</tr>
<tr>
<td>19</td>
<td>0.4913779068E+02</td>
<td>157.59</td>
</tr>
<tr>
<td>20</td>
<td>0.4902156685E+02</td>
<td>137.11</td>
</tr>
<tr>
<td>21</td>
<td>-0.4893605804E+02</td>
<td>116.60</td>
</tr>
</tbody>
</table>
Table 3.4: Edge voltages for a 50Ω microstrip line 0.8λ long at 10 GHz on GaAs substrate 0.127 mm thick with ε_r = 12.9. The port width is 0.057λ (14 ports along each edge).

### ELEMENTS OF MATRIX ZC

#### ELEMENTS OF COLUMN# 1

<table>
<thead>
<tr>
<th>ROW#</th>
<th>MAGNITUDE</th>
<th>DEG.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.5010519409E+02</td>
<td>0.37</td>
</tr>
<tr>
<td>2</td>
<td>0.4965384674E+02</td>
<td>-56.46</td>
</tr>
<tr>
<td>3</td>
<td>0.4960771942E+02</td>
<td>-71.11</td>
</tr>
<tr>
<td>4</td>
<td>0.4958567429E+02</td>
<td>-85.76</td>
</tr>
<tr>
<td>5</td>
<td>0.4957726288E+02</td>
<td>-100.39</td>
</tr>
<tr>
<td>6</td>
<td>0.4957650375E+02</td>
<td>-114.99</td>
</tr>
<tr>
<td>7</td>
<td>0.4956898499E+02</td>
<td>-129.57</td>
</tr>
<tr>
<td>8</td>
<td>0.4954758835E+02</td>
<td>-144.13</td>
</tr>
<tr>
<td>9</td>
<td>0.4950346756E+02</td>
<td>-158.68</td>
</tr>
<tr>
<td>10</td>
<td>0.4943912506E+02</td>
<td>-173.24</td>
</tr>
<tr>
<td>11</td>
<td>0.4935572433E+02</td>
<td>172.19</td>
</tr>
<tr>
<td>12</td>
<td>0.4926706696E+02</td>
<td>157.59</td>
</tr>
<tr>
<td>13</td>
<td>0.4917905807E+02</td>
<td>142.96</td>
</tr>
<tr>
<td>14</td>
<td>0.4910826111E+02</td>
<td>128.32</td>
</tr>
<tr>
<td>15</td>
<td>0.4905483627E+02</td>
<td>113.66</td>
</tr>
<tr>
<td>16</td>
<td>0.4965385056E+02</td>
<td>-56.46</td>
</tr>
<tr>
<td>17</td>
<td>0.4960771942E+02</td>
<td>-71.11</td>
</tr>
<tr>
<td>18</td>
<td>0.4958567429E+02</td>
<td>-85.76</td>
</tr>
<tr>
<td>19</td>
<td>0.4957725906E+02</td>
<td>-100.39</td>
</tr>
<tr>
<td>20</td>
<td>0.4957649994E+02</td>
<td>-114.99</td>
</tr>
<tr>
<td>21</td>
<td>0.4956898117E+02</td>
<td>-129.57</td>
</tr>
<tr>
<td>22</td>
<td>0.4954758453E+02</td>
<td>-144.13</td>
</tr>
<tr>
<td>23</td>
<td>0.4950346375E+02</td>
<td>-158.68</td>
</tr>
<tr>
<td>24</td>
<td>0.4943912125E+02</td>
<td>-173.24</td>
</tr>
<tr>
<td>25</td>
<td>0.4935572052E+02</td>
<td>172.19</td>
</tr>
<tr>
<td>26</td>
<td>0.4926706696E+02</td>
<td>157.59</td>
</tr>
<tr>
<td>27</td>
<td>0.4917905807E+02</td>
<td>142.96</td>
</tr>
<tr>
<td>28</td>
<td>0.4910825348E+02</td>
<td>128.32</td>
</tr>
<tr>
<td>29</td>
<td>0.4905484390E+02</td>
<td>113.66</td>
</tr>
</tbody>
</table>
Table 3.5: Edge voltages for a 50Ω microstrip line 0.8λ long at 10 GHz on GaAs substrate, 0.127 mm thick with ε_r = 12.9. The port width is 0.053λ. (15 ports along each edge)

**ELEMENTS OF MATRIX ZC**

**ELEMENTS OF COLUMN # 1**

<table>
<thead>
<tr>
<th>ROW#</th>
<th>MAGNITUDE</th>
<th>DEG.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.5010519409E+02</td>
<td>0.37</td>
</tr>
<tr>
<td>2</td>
<td>0.497751656E+02</td>
<td>-52.55</td>
</tr>
<tr>
<td>3</td>
<td>0.4974840927E+02</td>
<td>-59.39</td>
</tr>
<tr>
<td>4</td>
<td>0.4972575760E+02</td>
<td>-66.23</td>
</tr>
<tr>
<td>5</td>
<td>0.4970838547E+02</td>
<td>-73.07</td>
</tr>
<tr>
<td>6</td>
<td>0.4969667816E+02</td>
<td>-79.90</td>
</tr>
<tr>
<td>7</td>
<td>0.4968968201E+02</td>
<td>-86.74</td>
</tr>
<tr>
<td>8</td>
<td>0.4968567657E+02</td>
<td>-93.56</td>
</tr>
<tr>
<td>9</td>
<td>0.4968330765E+02</td>
<td>-100.39</td>
</tr>
<tr>
<td>10</td>
<td>0.4968185043E+02</td>
<td>-107.21</td>
</tr>
<tr>
<td>11</td>
<td>0.4968088913E+02</td>
<td>-114.02</td>
</tr>
<tr>
<td>12</td>
<td>0.4967943954E+02</td>
<td>-120.83</td>
</tr>
<tr>
<td>13</td>
<td>0.4967615128E+02</td>
<td>-127.63</td>
</tr>
<tr>
<td>14</td>
<td>0.4966960144E+02</td>
<td>-134.43</td>
</tr>
<tr>
<td>15</td>
<td>0.4965871429E+02</td>
<td>-141.22</td>
</tr>
<tr>
<td>16</td>
<td>0.4964304352E+02</td>
<td>-148.01</td>
</tr>
<tr>
<td>17</td>
<td>0.4962242508E+02</td>
<td>-154.80</td>
</tr>
<tr>
<td>18</td>
<td>0.4959701538E+02</td>
<td>-161.59</td>
</tr>
<tr>
<td>19</td>
<td>0.4956730652E+02</td>
<td>-168.39</td>
</tr>
<tr>
<td>20</td>
<td>0.4953359985E+02</td>
<td>-175.18</td>
</tr>
<tr>
<td>21</td>
<td>0.4949619675E+02</td>
<td>-178.02</td>
</tr>
<tr>
<td>22</td>
<td>0.4945553589E+02</td>
<td>-171.21</td>
</tr>
<tr>
<td>23</td>
<td>0.4941279984E+02</td>
<td>-164.40</td>
</tr>
<tr>
<td>24</td>
<td>0.4936999130E+02</td>
<td>-157.59</td>
</tr>
<tr>
<td>25</td>
<td>0.4932884598E+02</td>
<td>-150.77</td>
</tr>
<tr>
<td>26</td>
<td>0.4929013443E+02</td>
<td>-143.94</td>
</tr>
<tr>
<td>27</td>
<td>0.4925392914E+02</td>
<td>-137.11</td>
</tr>
<tr>
<td>28</td>
<td>0.4922046661E+02</td>
<td>-130.27</td>
</tr>
<tr>
<td>29</td>
<td>0.4919101334E+02</td>
<td>-123.43</td>
</tr>
<tr>
<td>30</td>
<td>0.4916714859E+02</td>
<td>-116.59</td>
</tr>
<tr>
<td>31</td>
<td>0.4914962387E+02</td>
<td>-109.75</td>
</tr>
</tbody>
</table>
10 GHz for two 0.4λ long sections of a 50Ω microstrip line, on a 127.1μm thick GaAs substrate with $\varepsilon_r = 12.9$, connected together by a number of interconnected ports. $Z_{11}$ results for different number of interconnected ports are given in Table 3.6. Referring to Table 3.6 one may conclude that more than four interconnected ports are needed to obtain accurate results. In computed results reported in this thesis usually 6 interconnected ports have been taken and the width of the interconnected port is less than 0.05 wavelength.

Table 3.6: Computed $Z_{11}$ for a 50Ω microstrip line on GaAs substrate for different numbers of interconnected ports.

<table>
<thead>
<tr>
<th>Number of Interconnected Ports</th>
<th>Computed $Z_{11}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>50.1051826</td>
</tr>
<tr>
<td>4</td>
<td>50.1055069</td>
</tr>
<tr>
<td>8</td>
<td>50.1056213</td>
</tr>
</tbody>
</table>

3.2.3 Radiating Region of a Discontinuity Configuration

In order to identify the regions of the discontinuity configuration that contribute dominantly to the power radiated, several computations were performed by taking different regions around the discontinuity. These computations show that in most of the cases the biggest contribution to the power radiated is from the ports in the region of the discontinuity itself. Results for a right-angled bend shown in Table 3.7 illustrate this point. The radiated power from a right-angled bend at 10 GHz as a function of the microstrip line length connected to each side of the right-angled bend on a substrate with $\varepsilon_r = 12.9$, ...
on 127.1\(\mu\)m thick GaAs substrate normalized with respect to the input power is listed in Table 3.7. One may note that most of the contribution to the power radiated is due to the voltage distribution around the corners of the right-angled bend.

Table 3.7: Radiated power as a function of the length of the microstrip line sections on each side of a right-angled bend.

\[
\begin{array}{|c|c|}
\hline
\text{Length X (shown in Figure 2.4) (Pr/Pl)dB} & \hline
0.4\lambda & -40.64 \\
0.35\lambda & -40.67 \\
0.3\lambda & -40.77 \\
0.25\lambda & -40.84 \\
0.2\lambda & -41.10 \\
\text{Ports on bend only} & -41.10 \\
(x = 1/2 line width) & \\
\hline
\end{array}
\]

3.3 Voltage Distribution Along the Edges of Microstrip Discontinuities

The multiport network model of the discontinuity configuration and the segmentation method are used to calculate voltage distribution around the edges of the discontinuity, in some typical cases. Edge voltage distribution is an intermediate step in our approach for radiation calculation. Also, study of edge voltage distribution helps one to identify the radiating regions of a discontinuity configuration.

3.3.1 Chamfered Right-Angled Bend

The voltage distribution at 10 GHz around the edges of a chamfered bend in 50\(\Omega\) microstrip line on 0.01 inch thick substrate with \(\varepsilon_r = 2.2\) is plotted
in Figure 3.1(a).

Phase values (in degrees) of the voltage distribution are plotted in Figure 3.1(b). One may note that far away from the junction the voltage values at the two edges of the microstrip line are equal. However, near to the discontinuity region the voltage distribution is asymmetric. As an example, at the first port on the left edge below the chamfered bend, the voltage is 49.8 V whereas the voltage in the right side edge is 50.7 V. The phase difference between these two voltages is 3.18 degrees.

It may be noted that the asymmetry in the voltage at the discontinuity region contributes to the radiated power from the microstrip discontinuity. When the voltages at the two edges of the microstrip line are equal, radiation from the two equivalent magnetic currents cancel each other. This argument may be extended to explain why a uniform, straight line does not radiate.

3.3.2 Right-Angled Bend (Unchamfered) on GaAs Substrate

The voltage distribution at 10 GHz around the edges of a right-angled bend in a 50Ω microstrip line on 0.1271mm thick substrate with εr = 12.9 is plotted in Figure 3.2(a). Phase values (in degrees) of the voltage distribution are plotted in 3.2(b). In this case as well, one may note that far away from the junction the voltage values are symmetric at the two edges of the microstrip line. However, near to the junction, the voltage distribution is asymmetric.

3.3.3 Right-Angled Bend (Unchamfered) on 0.254mm Thick Duroid Substrate

The voltage distribution around the edges of a right-angled bend in a 50Ω microstrip line on 0.254mm thick substrate with εr = 2.2 is plotted (at 10 GHz) in Figure 3.2(c). Phase values (in degrees) of the voltage distribution
Figure 3.1(a): Typical voltage distribution at the edges of a microstrip chamfered bend (voltage amplitude in volts). $\varepsilon_r = 2.2$; $h = 0.1$ inch; $Z_0 = 50\Omega$; $f = 10$ GHz.
Figure 3.1(b): Phase distribution of the edge voltages for a chamfered bend (phase values shown in degrees). $\varepsilon_r = 2.2; \ h = 0.01 \text{ inch}; \ Z_0 = 50\Omega; \ f = 10 \text{ GHz}$. 
Figure 3.2(a): Typical voltage distribution at the edges of a microstrip right-angled bend (voltage amplitude in volts). $\varepsilon_r = 12.9; h = 0.127\text{mm}; Z_0 = 50\Omega; f = 10\text{ GHz}.$
Figure 3.2(b): Phase distribution of the edge voltages for a right-angled bend (Phase values shown in degrees). \( \varepsilon_r = 12.9; \ k = 0.1271\text{mm}; \ Z_0 = 50\Omega; \ f = 10\ \text{GHz}. \)
Figure 3.2(c): Typical voltage distribution at the edges of a microstrip right-angled bend (voltage amplitude in volts) $\varepsilon_r = 2.2$; $h = 10\text{mil}$; $Z_0 = 50\Omega$; $f = 10\text{ GHz}$. 
Figure 3.2(d): Phase distribution of the edge voltage for a right-angled bend (phase values in degrees) $\varepsilon_r = 2.2; \ h = 10\text{mil}; \ Z_0 = 50\Omega; \ f = 10\text{GHz}$. 
are shown in Figure 3.2(d). In this case as well one may note that far away from the junction the voltage is symmetric at the two edges of the microstrip line. However, near to the junction the voltage distribution is asymmetric. Also, we note by comparison of Figures 3.2(a) and 3.2(c) that when the circuit is fabricated on a substrate with a lower dielectric constant, the asymmetry in the voltage distribution along the discontinuity edges is more significant. Numerical results show that the asymmetry in the voltage distribution near the junction is enhanced when the right-angled bend in a 50Ω microstrip line is fabricated on a thicker substrate.

3.3.4 Step Junction

The voltage distribution at the edges of a step junction at 30 GHz when the input power is fed from a 70.7Ω line on a substrate with εr = 2.2 and thickness 10 mil is plotted in Figure 3.3. One may note that the ports at the step junction have a significant contribution to the power radiated from a step discontinuity.

3.3.5 Stub

The voltage distribution at the edges of a 20Ω stub at 11 GHz on a substrate with εr = 2.2, 10 mil thick, is plotted in Figure 3.4. Voltage levels at the various edges in the circuit when excited by a unit (1 Amp) current (with source impedance of 50Ω) at the input port with the other port terminated with a 50Ω load are also shown in the figure. This voltage distribution has been calculated at 11 GHz where the stub length is approximately a quarter wavelength. As expected, the maximum voltage appears at the open end of the stub and decreases monotonically to small values at the junction of the stub and the main line. This happens because the stub presents a virtual short at the main line, causing most of the power to be reflected back, and the voltage
Figure 3.3: Voltage distribution at the edges of a step junction when the input power is fed from a 70.7Ω line. \( \varepsilon_r = 2.2; h = 0.01 \) mil; \( Z_1 = 70.7\Omega; Z_2 = 50\Omega \) \( f = 10 \) GHz. Voltage magnitude in volts, phase angle in degrees.
values on the line near to the matched load are much smaller.

3.4 Radiation from a Right-Angled Microstrip Bend

Results for the power radiated by the 90° bends (for 20Ω and 50Ω lines on \( \varepsilon_r = 2.2, h = 10 \) mil, and for 50Ω bend on 127.1\( \mu \)m thick GaAs substrate) normalized with respect to the input power are shown in Figure 3.5 for frequencies ranging from 10 GHz to 40 GHz. These results are in good agreement with results based on the complex Poynting vector method [6], which are also plotted in Figure 3.5. The radiation loss at 40 GHz is 0.0002 dB for a right-angled bend in an 50Ω line on (\( \varepsilon_r = 12.9 \)) GaAs substrate, 127.1\( \mu \)m thick. It is important to know beyond what frequency range, the radiation loss from circuits printed on GaAs cannot be neglected. At 90 GHz the radiation loss for a right-angled bend in a 50Ω line bend on GaAs (\( \varepsilon_r = 12.9 \)) 127.1\( \mu \)m thick is 0.077 dB. Radiated power levels for frequencies from 10 GHz to 90 GHz are plotted in Figure 3.6 for a 50Ω bend on GaAs 127.1\( \mu \)m thick. Results obtained by the present approach (MN model) are compared with the Poynting vector (PV method). The values of radiation loss from a 90° bend in 50Ω line on a substrate with \( \varepsilon_r = 2.2 \) at 30 GHz and at 40 GHz are 0.1 dB and 0.17 dB, respectively. These results point out that in some circuit configuration radiation from bends may need to be taken into account for accurate design.

3.5 Radiation from a Chamfered Bend

The configuration analyzed consists of two chamfered bends in a 50Ω line connected to a 50Ω line at the bend corners on a 0.01 inch thick substrate with \( \varepsilon_r = 2.2 \), as shown in Figure 3.7(a). The distance between the centers of the two chamfered bends is quarter wavelength at 10 GHz.

In order to calculate voltages around the edges of the circuit the config-
Figure 3.4: Voltage distribution at the edges of a 20Ω open-end stub.
3.5 Normalized radiated power from a right-angled bend (MNM: Multiport Network Model and PV: Poynting Vector Method).

ulation is broken down into seven regular segments (including two resistances at input and output ports) as shown in Figure 3.7(b). Segment number one is a matched source and the last segment is a matched $50\Omega$ load. Segments 2, 4 and 6 are rectangular segments and segments 3 and 5 are right-angle isosceles triangular segments. Single summation formulations [34,35] for the Green's functions are used. Results for the power radiated by this circuit normalized with respect to the input power are shown in Figure 3.8 for frequencies from 8 GHz to 12 GHz. We note that radiated power increases linearly with frequency. The radiated power at 10 GHz is -28.1 dB. One may note that radiation loss from a chamfered bend are of the same order as from a right-angled bend (in which case the corresponding value is -26.5 dB).

3.6 Radiation from a Compensated Bend with a Change in Line Impedances

The configuration analyzed consists of two compensated bends in a $70\Omega$ line connected to a $50\Omega$ line on 10 mil thick substrate with $\varepsilon_r = 2.2$ as shown in
Figure 3.6: Radiated power from bend discontinuity on GaAs substrate.
Figure 3.7: Multiple ports along the edges of a chamfered bend discontinuity.
(a) Multiport network model for chamfered bend circuit.
(b) Regular segments for the chamfered bend circuit.

Figure 3.9(a). The distance between the centers of the two bends is a quarter wavelength at 10 GHz.

To evaluate voltages around the edges of the circuit, the configuration is broken down into eleven regular segments as shown in Figure 3.9(b). The first segment is a matched source and the last segment is a matched load. Segments 2, 3, 5, 6, 7, 9 and 10 are rectangular segments and segments 4 and 8 are right-angle isosceles triangle segments.

Results for the power radiated by this circuit normalized with respect to the input power are shown in Figure 3.10 for frequencies ranging from 8 GHz to 12 GHz. The radiated power at 12 GHz is -21.75 dB which is 4.68
Figure 3.8: Power radiated from a chamfered bend.

dB higher than the power radiated by the chamfered bend circuit described in Section 3.5. One may note that the power radiated from a compensated bend with change in width is higher (by around 5 dB) than the power radiated from a chamfered bend with equal impedance lines on two sides. Also, the variation of the radiated power with frequency is no more linear.

3.7 Radiation from a Microstrip Step Junction

Power radiated from a step junction discontinuity produced by a change in impedance from $50\Omega$ to $10\Omega$, on $\varepsilon_r = 2.2$ substrate with thickness of $0.79\text{mm}$ is plotted in Figure 3.11. The radiation loss at 10 GHz when the input power is fed from the $50\Omega$ line is 0.7 dB. A similar computation at 30 GHz for a $50\Omega$ to
Figure 3.9: Multiport network modeling of a compensated bend.
(a) Multiple ports at the edges of a circuit with two compensated bends.
(b) A circuit with two compensated bends decomposed into regular segments.

Figure 3.10: Radiated power from a compensated bend on substrate with $\varepsilon_r = 2.2$, 10 mil thick.
70.7Ω junction (on ε_r = 2.2, 0.01 inch thick), yields the normalized radiated power to be -24.8 dB when the input power is fed from the 50Ω line, and -33 dB when the power is fed from the 70.7Ω line. These numbers correspond to radiation loss values of 0.0143 dB and 0.00217 dB respectively. This asymmetric nature of radiation from a step discontinuity depending on the feed port is understandable. When the input wave is incident from the wider line, the reflection from the junction produces fringing fields similar to that at the radiating edge of a rectangular microstrip patch antenna. Hence, increased radiation loss in this case.

![Diagram of a 70.7Ω junction with parameters and radiation power vs. frequency graph.](image)

Figure 3.11: Normalized radiated power from a step (MNM: Multiport Network Model and PV: Poynting Vector Method).

### 3.8 Radiation from a T-junction Discontinuity

Computed results for power radiated from a T-junction (50Ω main line with a 35.35Ω branch line on a substrate with ε_r = 2.2, 0.79mm thick) are shown in Figure 3.12. The radiation loss for the input power incident from a branch line is 0.18 dB at 12 GHz. It is found that for such a T-junction most of the
contribution to the radiation loss originates from the region of the junction. The Poynting vector method results are also shown in this figure. It may be noted that radiation from a T-junction is also significant enough to be taken into account. This becomes very relevant when a large number of T-junctions are used such as in a corporate-feed arrangement for feeding arrays of microstrip patch antennas.

![Diagram](image)

Figure 3.12: Normalized radiated power from a T-junction. (MNM: Multiport Network Model and PV: Poynting Vector Method).

3.9 Radiation from a 20Ω Stub

Power radiated from a 20Ω stub, quarter wavelength at 10 GHz, connected to a 50Ω line on a εr = 2.2 substrate with thickness of 0.01 inch is plotted in Figure 3.13. The physical length of the stub is a multiple of quarter wavelength at 10 GHz and at 30 GHz. Because of open-end and junction effects, the resonance frequencies (which correspond to peaks of the radiated power) are shifted to 11.3 GHz and 33 GHz respectively. The radiated power at 11.3
GHz is -1.6 dB and at 33 GHz is -0.9 dB. Since the radiated power is high in this case, we used the iterative procedure described in Subsection 2.3.3 to check the convergence of edge voltages. It may be noted that radiation from a 20Ω, quarter wavelength open-end stub is significant and should be taken into account in design of microstrip circuits and antennas.

![Figure 3.13: Normalized radiated power from a 20Ω open-end stub (MNM: Multiport Network Model and PV: Poynting Vector Method).](image_url)

3.10 Radiation from a Gap Discontinuity

S-parameter numerical results of a gap discontinuity using fullwave analysis approach include radiation loss, but results for power radiation from a gap discontinuity are not available in literature. Multiport network model has been employed to evaluate radiation from gap discontinuities also. Electric field coupling in the gap region is modeled by a lumped network and is derived from the capacitance matrix of gap discontinuity. Capacitances for gap discontinuity are obtained from equations given in [37]. A π-network representation of the
The $[C]$ matrix is shown in Figure 3.14(a). Capacitances $C_{11}$ and $C_{22}$ represent the edge capacitances of microstrip lines 1 and 2 respectively. $\Delta l$ is the width of port $i$ (see Figure 3.15). The network section shown in Figure 3.14(b) may be represented by the following $Y$-matrix:

$$
[Y_C] = j\omega \begin{bmatrix} C_1 + C_g & -C_g \\ -C_g & C_2 + C_g \end{bmatrix} = j\omega [C_g]
$$

(3.1)

The multiport network model for gap discontinuity is shown in Figure 3.15. When the lumped network has $n$ ports on each side, the complete $C$-matrix may be written as shown in equation (3.2). All non-diagonal terms in each of the four submatrices are zeros.

![Figure 3.14: $\pi$-network representation of the $[C]$ matrix.](image)
The corresponding impedance matrix may be written as

\[
Z_G = Y_G^{-1} = (j\omega C_g)^{-1}
\]  

(3.3)

Power radiated from an asymmetric gap discontinuity (change in impedance from 48Ω to 32.8Ω, the width of the gap is equal to the height of the substrate 0.635mm, \( \varepsilon_r = 10.4 \)) is plotted in Figure 3.16. The radiation loss values (for the input power incident from the 48Ω line) are 0.164 dB and 0.785 dB at 4 GHz and 12 GHz, respectively. It was found that the radiation loss from asymmetric gap and symmetric gap are almost the same since the contribution of the magnetic current element at the edge of the input line in the gap region is dominant.

3.11 Radiation from Asymmetric Step Discontinuity when Higher-Order Modes are Excited

As discussed earlier in Section 2.2, the planar waveguide model for microstrip lines is valid [28] for higher-order modes also. To evaluate power radiated from microstrip discontinuities when higher-order modes are also ex-
Figure 3.15: Multiport network model for a gap discontinuity.

cited, the radiated power from two asymmetric step discontinuities in cascade, i.e., from 50Ω to 20Ω and again to 50Ω is considered. The configuration analyzed is shown in Figure 3.16, and is decomposed into three rectangular segments: 50Ω, 20Ω and 50Ω microstrip lines on a substrate with $\varepsilon_r = 2.2$, 20 mil thick.

The cutoff frequencies of the higher-order modes are given by

$$f_c(m, 0) = \frac{m c_0}{2 \sqrt{\varepsilon_r(f) \varepsilon(f)}}$$

(3.4)

where $m$ is the mode order and $c_0$ is the velocity of electromagnetic waves in free space. For the 20Ω microstrip line on a substrate with $\varepsilon_r = 2.2$, 20 mil thick, the first higher order mode is excited around 16 GHz.

The discontinuity configuration and ports that have been considered
for evaluating radiation loss are shown in Figure 3.17. Fifty-seven open ports were added at the edges of the discontinuity. Radiation conductance networks were added to ports two through fifty-seven, along the edges as described in Section 2.3 and iterative computations were carried out for evaluating voltage distribution along the edges. The voltage distributions for the configuration shown in Figure 3.17, at 15 GHz, 16 GHz and 17 GHz are given in Tables 3.8 to 3.10. Comparison of Tables 3.8 and 3.9 shows that edge voltages at 15 GHz and 17 GHz are almost around the same level, however the magnitude of edge voltages at 16 GHz are higher than these at 15 GHz and 17 GHz. Results for the power radiated by this circuit normalized with respect to the input power are shown in Figure 3.17 for frequency range from 14 GHz to 17 GHz. The radiated power at 16 GHz is -3.1 dB and it decreases to -4.1 dB at 17 GHz.

As shown in Tables 3.8 and 3.9 the voltage distribution along the circuit edges at 15 GHz and 17 GHz is around the same level, and as shown in Figure 3.18 the power radiated at 15 GHz and 17 GHz is approximately -4.1 dB. However, at 16 GHz the magnitude of voltages along the circuit edges are higher than at 17 GHz and so power radiated at 16 GHz is higher than power radiated at 17 GHz.

3.12 Comparison of Power Radiated Results With Fullwave Analysis Results

Fullwave analysis has been used [18-20] to evaluate spurious coupling among microstrip discontinuities. In the fullwave analysis approach a mixed potential integral equation is solved to characterize the fields everywhere for a microstrip structure. In this approach, effects such as radiation and mutual coupling are included in the analysis by using the exact Green's functions for microstrip structures. The two-dimensional surface current on the microstrip
Figure 3.16: Power radiated from asymmetric gap.
Figure 3.17: Multiple ports for an asymmetric step discontinuity from 50Ω to 20Ω.
Figure 3.18: Power radiated from asymmetric step when higher-order modes are excited.
Table 3.8: Voltage distribution at 15 GHz along the edges of an asymmetric step discontinuity.

<table>
<thead>
<tr>
<th>ROW #</th>
<th>MAGNITUDE</th>
<th>DEG.</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.13070E+02</td>
<td>42.87</td>
</tr>
<tr>
<td>3</td>
<td>0.87225E+01</td>
<td>-90.50</td>
</tr>
<tr>
<td>4</td>
<td>0.26538E+02</td>
<td>-111.50</td>
</tr>
<tr>
<td>5</td>
<td>0.40072E+02</td>
<td>-115.85</td>
</tr>
<tr>
<td>6</td>
<td>0.46525E+02</td>
<td>-118.17</td>
</tr>
<tr>
<td>7</td>
<td>0.45628E+02</td>
<td>-120.03</td>
</tr>
<tr>
<td>8</td>
<td>0.13122E+02</td>
<td>42.93</td>
</tr>
<tr>
<td>9</td>
<td>0.86192E+01</td>
<td>-90.07</td>
</tr>
<tr>
<td>10</td>
<td>0.26197E+02</td>
<td>-11.36</td>
</tr>
<tr>
<td>11</td>
<td>0.39062E+02</td>
<td>-115.70</td>
</tr>
<tr>
<td>12</td>
<td>0.43529E+02</td>
<td>-117.94</td>
</tr>
<tr>
<td>13</td>
<td>0.36352E+02</td>
<td>-119.78</td>
</tr>
<tr>
<td>14</td>
<td>0.71962E+01</td>
<td>-126.15</td>
</tr>
<tr>
<td>15</td>
<td>0.12532E+02</td>
<td>61.94</td>
</tr>
<tr>
<td>16</td>
<td>0.26015E+02</td>
<td>59.68</td>
</tr>
<tr>
<td>17</td>
<td>0.33113E+02</td>
<td>58.94</td>
</tr>
<tr>
<td>18</td>
<td>0.39428E+02</td>
<td>-122.01</td>
</tr>
<tr>
<td>19</td>
<td>0.30709E+02</td>
<td>-124.52</td>
</tr>
<tr>
<td>20</td>
<td>0.21900E+02</td>
<td>-128.16</td>
</tr>
<tr>
<td>21</td>
<td>0.14668E+02</td>
<td>-133.59</td>
</tr>
<tr>
<td>22</td>
<td>0.98938E+01</td>
<td>-140.60</td>
</tr>
<tr>
<td>23</td>
<td>0.77535E+01</td>
<td>-145.06</td>
</tr>
<tr>
<td>24</td>
<td>0.79199E+01</td>
<td>-141.93</td>
</tr>
<tr>
<td>25</td>
<td>0.99218E+01</td>
<td>-135.24</td>
</tr>
<tr>
<td>26</td>
<td>0.12928E+02</td>
<td>-130.42</td>
</tr>
<tr>
<td>27</td>
<td>0.15803E+02</td>
<td>-128.08</td>
</tr>
<tr>
<td>28</td>
<td>0.17446E+02</td>
<td>-127.51</td>
</tr>
<tr>
<td>29</td>
<td>0.17030E+02</td>
<td>-128.30</td>
</tr>
<tr>
<td>30</td>
<td>0.34442E+02</td>
<td>58.86</td>
</tr>
<tr>
<td>31</td>
<td>0.35015E+02</td>
<td>58.99</td>
</tr>
<tr>
<td>32</td>
<td>0.35568E+02</td>
<td>58.84</td>
</tr>
<tr>
<td>33</td>
<td>0.35246E+02</td>
<td>58.36</td>
</tr>
<tr>
<td>34</td>
<td>0.33318E+02</td>
<td>57.47</td>
</tr>
<tr>
<td>35</td>
<td>0.29427E+02</td>
<td>55.96</td>
</tr>
<tr>
<td>36</td>
<td>0.23708E+02</td>
<td>53.36</td>
</tr>
<tr>
<td>37</td>
<td>0.16784E+02</td>
<td>48.29</td>
</tr>
<tr>
<td>38</td>
<td>0.97613E+01</td>
<td>35.89</td>
</tr>
<tr>
<td>39</td>
<td>0.48687E+01</td>
<td>-5.43</td>
</tr>
<tr>
<td>40</td>
<td>0.56994E+01</td>
<td>-64.44</td>
</tr>
<tr>
<td>41</td>
<td>0.77409E+01</td>
<td>-81.62</td>
</tr>
<tr>
<td>42</td>
<td>0.13020E+02</td>
<td>-116.78</td>
</tr>
<tr>
<td>43</td>
<td>0.10818E+02</td>
<td>-105.99</td>
</tr>
<tr>
<td>44</td>
<td>0.90772E+01</td>
<td>-94.00</td>
</tr>
<tr>
<td>45</td>
<td>0.82628E+01</td>
<td>-85.36</td>
</tr>
<tr>
<td>46</td>
<td>0.14614E+02</td>
<td>-130.58</td>
</tr>
<tr>
<td>47</td>
<td>0.12124E+02</td>
<td>-134.75</td>
</tr>
<tr>
<td>48</td>
<td>0.99292E+01</td>
<td>-141.13</td>
</tr>
<tr>
<td>49</td>
<td>0.79800E+01</td>
<td>-150.72</td>
</tr>
<tr>
<td>50</td>
<td>0.63894E+01</td>
<td>-165.40</td>
</tr>
<tr>
<td>51</td>
<td>0.54351E+01</td>
<td>173.09</td>
</tr>
<tr>
<td>52</td>
<td>0.13541E+02</td>
<td>-127.77</td>
</tr>
<tr>
<td>53</td>
<td>0.11784E+02</td>
<td>-133.91</td>
</tr>
<tr>
<td>54</td>
<td>0.98181E+01</td>
<td>-140.89</td>
</tr>
<tr>
<td>55</td>
<td>0.79437E+01</td>
<td>-150.66</td>
</tr>
<tr>
<td>56</td>
<td>0.63716E+01</td>
<td>-165.41</td>
</tr>
<tr>
<td>57</td>
<td>0.54308E+01</td>
<td>173.07</td>
</tr>
</tbody>
</table>
Table 3.9: Voltage distribution at 16 GHz along the edges of an asymmetric step discontinuity.

<table>
<thead>
<tr>
<th>ROW #</th>
<th>MAGNITUDE</th>
<th>DEG.</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.25508E+03</td>
<td>28.46</td>
</tr>
<tr>
<td>3</td>
<td>0.18754E+03</td>
<td>22.19</td>
</tr>
<tr>
<td>4</td>
<td>0.83276E+02</td>
<td>2.63</td>
</tr>
<tr>
<td>5</td>
<td>0.71530E+02</td>
<td>-113.60</td>
</tr>
<tr>
<td>6</td>
<td>0.18869E+03</td>
<td>-138.69</td>
</tr>
<tr>
<td>7</td>
<td>0.29449E+03</td>
<td>-145.53</td>
</tr>
<tr>
<td>8</td>
<td>0.25578E+03</td>
<td>28.46</td>
</tr>
<tr>
<td>9</td>
<td>0.18910E+03</td>
<td>22.25</td>
</tr>
<tr>
<td>10</td>
<td>0.87250E+02</td>
<td>3.93</td>
</tr>
<tr>
<td>11</td>
<td>0.61861E+02</td>
<td>-106.43</td>
</tr>
<tr>
<td>12</td>
<td>0.15256E+03</td>
<td>-135.75</td>
</tr>
<tr>
<td>13</td>
<td>0.18300E+03</td>
<td>-142.31</td>
</tr>
<tr>
<td>14</td>
<td>0.42973E+02</td>
<td>12.25</td>
</tr>
<tr>
<td>15</td>
<td>0.29378E+03</td>
<td>26.44</td>
</tr>
<tr>
<td>16</td>
<td>0.49595E+03</td>
<td>27.05</td>
</tr>
<tr>
<td>17</td>
<td>0.60908E+03</td>
<td>27.16</td>
</tr>
<tr>
<td>18</td>
<td>0.38240E+03</td>
<td>-149.54</td>
</tr>
<tr>
<td>19</td>
<td>0.45908E+03</td>
<td>-152.44</td>
</tr>
<tr>
<td>20</td>
<td>0.53135E+03</td>
<td>-154.57</td>
</tr>
<tr>
<td>21</td>
<td>0.59647E+03</td>
<td>-156.01</td>
</tr>
<tr>
<td>22</td>
<td>0.64659E+03</td>
<td>-156.89</td>
</tr>
<tr>
<td>23</td>
<td>0.67358E+03</td>
<td>-157.37</td>
</tr>
<tr>
<td>24</td>
<td>0.67236E+03</td>
<td>-157.58</td>
</tr>
<tr>
<td>25</td>
<td>0.64250E+03</td>
<td>-157.66</td>
</tr>
<tr>
<td>26</td>
<td>0.58834E+03</td>
<td>-157.74</td>
</tr>
<tr>
<td>27</td>
<td>0.51743E+03</td>
<td>-157.98</td>
</tr>
<tr>
<td>28</td>
<td>0.43736E+03</td>
<td>-158.59</td>
</tr>
<tr>
<td>29</td>
<td>0.35122E+03</td>
<td>-159.83</td>
</tr>
<tr>
<td>30</td>
<td>0.62563E+03</td>
<td>27.18</td>
</tr>
<tr>
<td>31</td>
<td>0.60758E+03</td>
<td>27.25</td>
</tr>
<tr>
<td>32</td>
<td>0.57580E+03</td>
<td>27.29</td>
</tr>
<tr>
<td>33</td>
<td>0.53804E+03</td>
<td>27.13</td>
</tr>
<tr>
<td>34</td>
<td>0.50365E+03</td>
<td>26.55</td>
</tr>
<tr>
<td>35</td>
<td>0.48112E+03</td>
<td>25.43</td>
</tr>
<tr>
<td>36</td>
<td>0.47582E+03</td>
<td>23.83</td>
</tr>
<tr>
<td>37</td>
<td>0.48843E+03</td>
<td>22.08</td>
</tr>
<tr>
<td>38</td>
<td>0.51470E+03</td>
<td>20.53</td>
</tr>
<tr>
<td>39</td>
<td>0.54661E+03</td>
<td>19.37</td>
</tr>
<tr>
<td>40</td>
<td>0.57476E+03</td>
<td>18.66</td>
</tr>
<tr>
<td>41</td>
<td>0.59112E+03</td>
<td>18.33</td>
</tr>
<tr>
<td>42</td>
<td>0.41701E+02</td>
<td>6.73</td>
</tr>
<tr>
<td>43</td>
<td>0.27714E+03</td>
<td>16.91</td>
</tr>
<tr>
<td>44</td>
<td>0.46808E+03</td>
<td>17.92</td>
</tr>
<tr>
<td>45</td>
<td>0.57530E+03</td>
<td>18.23</td>
</tr>
<tr>
<td>46</td>
<td>0.26332E+03</td>
<td>-162.16</td>
</tr>
<tr>
<td>47</td>
<td>0.20224E+03</td>
<td>-166.17</td>
</tr>
<tr>
<td>48</td>
<td>0.15981E+03</td>
<td>-172.25</td>
</tr>
<tr>
<td>49</td>
<td>0.12599E+03</td>
<td>178.50</td>
</tr>
<tr>
<td>50</td>
<td>0.99106E+02</td>
<td>163.91</td>
</tr>
<tr>
<td>51</td>
<td>0.82782E+02</td>
<td>141.65</td>
</tr>
<tr>
<td>52</td>
<td>0.16720E+03</td>
<td>-162.48</td>
</tr>
<tr>
<td>53</td>
<td>0.17263E+03</td>
<td>-166.93</td>
</tr>
<tr>
<td>54</td>
<td>0.15107E+03</td>
<td>-172.88</td>
</tr>
<tr>
<td>55</td>
<td>0.12338E+03</td>
<td>178.06</td>
</tr>
<tr>
<td>56</td>
<td>0.98377E+02</td>
<td>163.62</td>
</tr>
<tr>
<td>57</td>
<td>0.82589E+02</td>
<td>141.44</td>
</tr>
</tbody>
</table>
Table 3.10: Voltage distribution at 17 GHz along the edges of an asymmetric step discontinuity.

<table>
<thead>
<tr>
<th>ROW #</th>
<th>MAGNITUDE</th>
<th>DEG.</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.3656E+02</td>
<td>19.68</td>
</tr>
<tr>
<td>3</td>
<td>0.18524E+02</td>
<td>13.62</td>
</tr>
<tr>
<td>4</td>
<td>0.5534E+01</td>
<td>-123.06</td>
</tr>
<tr>
<td>5</td>
<td>0.2653E+02</td>
<td>-151.82</td>
</tr>
<tr>
<td>6</td>
<td>0.4199E+02</td>
<td>-155.39</td>
</tr>
<tr>
<td>7</td>
<td>0.4786E+02</td>
<td>-157.25</td>
</tr>
<tr>
<td>8</td>
<td>0.36599E+02</td>
<td>19.68</td>
</tr>
<tr>
<td>9</td>
<td>0.18592E+02</td>
<td>13.64</td>
</tr>
<tr>
<td>10</td>
<td>0.53812E+01</td>
<td>-121.87</td>
</tr>
<tr>
<td>11</td>
<td>0.25998E+02</td>
<td>-151.68</td>
</tr>
<tr>
<td>12</td>
<td>0.40433E+02</td>
<td>-155.27</td>
</tr>
<tr>
<td>13</td>
<td>0.43152E+02</td>
<td>-157.16</td>
</tr>
<tr>
<td>14</td>
<td>0.28917E+02</td>
<td>-159.14</td>
</tr>
<tr>
<td>15</td>
<td>0.19161E+02</td>
<td>-159.75</td>
</tr>
<tr>
<td>16</td>
<td>0.12684E+02</td>
<td>-159.60</td>
</tr>
<tr>
<td>17</td>
<td>0.93332E+01</td>
<td>-159.17</td>
</tr>
<tr>
<td>18</td>
<td>0.43332E+02</td>
<td>-158.87</td>
</tr>
<tr>
<td>19</td>
<td>0.30638E+02</td>
<td>-161.19</td>
</tr>
<tr>
<td>20</td>
<td>0.12524E+02</td>
<td>-169.13</td>
</tr>
<tr>
<td>21</td>
<td>0.78366E+01</td>
<td>42.92</td>
</tr>
<tr>
<td>22</td>
<td>0.24538E+02</td>
<td>28.88</td>
</tr>
<tr>
<td>23</td>
<td>0.35304E+02</td>
<td>26.72</td>
</tr>
<tr>
<td>24</td>
<td>0.38675E+02</td>
<td>26.23</td>
</tr>
<tr>
<td>25</td>
<td>0.34697E+02</td>
<td>26.71</td>
</tr>
<tr>
<td>26</td>
<td>0.25363E+02</td>
<td>28.65</td>
</tr>
<tr>
<td>27</td>
<td>0.13913E+02</td>
<td>34.81</td>
</tr>
<tr>
<td>28</td>
<td>0.44712E+01</td>
<td>67.91</td>
</tr>
<tr>
<td>29</td>
<td>0.48216E+01</td>
<td>160.64</td>
</tr>
<tr>
<td>30</td>
<td>0.77985E+01</td>
<td>-158.75</td>
</tr>
<tr>
<td>31</td>
<td>0.24221E+01</td>
<td>-152.79</td>
</tr>
<tr>
<td>32</td>
<td>0.65106E+01</td>
<td>15.11</td>
</tr>
<tr>
<td>33</td>
<td>0.21861E+02</td>
<td>15.95</td>
</tr>
<tr>
<td>34</td>
<td>0.22261E+02</td>
<td>14.12</td>
</tr>
<tr>
<td>35</td>
<td>0.13698E+02</td>
<td>8.61</td>
</tr>
<tr>
<td>36</td>
<td>0.44858E+01</td>
<td>40.72</td>
</tr>
<tr>
<td>37</td>
<td>0.15777E+02</td>
<td>-143.00</td>
</tr>
<tr>
<td>38</td>
<td>0.33336E+02</td>
<td>-151.17</td>
</tr>
<tr>
<td>39</td>
<td>0.33336E+02</td>
<td>-153.56</td>
</tr>
<tr>
<td>40</td>
<td>0.56022E+02</td>
<td>-154.51</td>
</tr>
<tr>
<td>41</td>
<td>0.26735E+02</td>
<td>-158.66</td>
</tr>
<tr>
<td>42</td>
<td>0.40288E+02</td>
<td>-155.99</td>
</tr>
<tr>
<td>43</td>
<td>0.30708E+02</td>
<td>-154.98</td>
</tr>
<tr>
<td>44</td>
<td>0.56450E+02</td>
<td>-154.70</td>
</tr>
<tr>
<td>45</td>
<td>0.67697E+01</td>
<td>176.07</td>
</tr>
<tr>
<td>46</td>
<td>0.67537E+01</td>
<td>176.83</td>
</tr>
<tr>
<td>47</td>
<td>0.58827E+01</td>
<td>172.59</td>
</tr>
<tr>
<td>48</td>
<td>0.47728E+01</td>
<td>164.15</td>
</tr>
<tr>
<td>49</td>
<td>0.37471E+01</td>
<td>149.73</td>
</tr>
<tr>
<td>50</td>
<td>0.30894E+01</td>
<td>126.65</td>
</tr>
<tr>
<td>51</td>
<td>0.11901E+02</td>
<td>-168.77</td>
</tr>
<tr>
<td>52</td>
<td>0.82216E+01</td>
<td>-176.87</td>
</tr>
<tr>
<td>53</td>
<td>0.63090E+01</td>
<td>175.40</td>
</tr>
<tr>
<td>54</td>
<td>0.48897E+01</td>
<td>165.49</td>
</tr>
<tr>
<td>55</td>
<td>0.37741E+01</td>
<td>150.40</td>
</tr>
<tr>
<td>56</td>
<td>0.30924E+01</td>
<td>127.03</td>
</tr>
</tbody>
</table>
configuration is found by enforcing the boundary conditions and then applying the method of moments to the integral equation. The integral equation is reduced to a matrix equation and the unknown currents are solved by using LU decomposition and back substitutions. The effect of the junction is parameterized by probing the current away from the junction. The P-Mesh code [18] is a fullwave solver for analyzing passive microstrip discontinuities. The surface current on the microstrip configuration is found by discretizing the discontinuity configuration into elementary cells and assuming that the current in each cell has a linear distribution with unknown coefficients. This procedure reduces the integral equation to an algebraic matrix equation. The direct solution of P-Mesh equations yield the surface currents on the microstrip. Additional computations are required to extract or de-embed the scattering parameters of the discontinuity from the computed current values. P-Mesh extracts the S-parameters by probing the currents on the feed strips away from the reference plane of the discontinuity. Therefore, for a given discontinuity, additional strips must be added to each port of the structure for probing purposes.

The power radiated from a discontinuity may be evaluated from the S-parameters obtained by using fullwave analysis, of a given discontinuity configuration, by considering a lossless case. For a lossless junction the total power leaving the $N$ ports must equal the total incident power. The power-conservation condition may be expressed as

$$\sum_{n=1}^{N} S_{ni} S_{ni}^* = 1$$

(3.5)

The index $i$ is arbitrary; equation 3.5 must hold for all values of $i$. Equation 3.5 states that for a lossless junction the product of any column of the scattering matrix with the conjugate of this same column equals unity.
The normalized power radiated may be evaluated from the $S$-parameters, obtained by using fullwave analysis, as

$$\frac{P_r}{P_i} = 1 - \sum_{n=1}^{N} S_{ni} S_{ni}^*$$  \hfill (3.6)

**Power radiated from an open end**

Results of power radiated from a 20Ω open end on $\varepsilon_r = 2.2$ substrate 10 mil thick, for frequencies ranging from 10 GHz to 40 GHz are shown in Figure 3.20. Power radiated results obtained by using the MNM are plotted as curve 2, power radiated results obtained by using the Poynting vector method are plotted as curve 1. Power radiated results obtained by using the P-Mesh code are plotted as curve 3. $S_{11}$ results obtained by using the P-Mesh code are listed in Table 3.11. One may note that there is good agreement between power radiated results using the MNM and fullwave analysis (P-Mesh code).

**Table 3.11: $S_{11}$ results for a 20Ω open end on $\varepsilon_r = 2.2$ substrate, 10 mil thick, using P-Mesh Code.**

<table>
<thead>
<tr>
<th>Frequency (GHz)</th>
<th>$S_{11}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>1.0000068 $\angle$ -5.65°</td>
</tr>
<tr>
<td>20</td>
<td>0.986427 $\angle$ -11.47°</td>
</tr>
<tr>
<td>30</td>
<td>0.977956 $\angle$ -16.12°</td>
</tr>
<tr>
<td>40</td>
<td>0.973599 $\angle$ -19.37°</td>
</tr>
</tbody>
</table>

**Power radiated from a right-angled bend**

Comparison of power-radiated results from a 50Ω right-angled bend on $\varepsilon_r = 2.2$ substrate, 10 mil thick, for frequencies ranging from 10 GHz to 40 GHz are shown in Figure 3.20. Power radiated results obtained by using the PV
method are plotted as curve 1. Power radiated results obtained by using the MNM are plotted as curve 2. Power radiated results obtained by using the P-Mesh code when the de-embedding section is 0.7λ and 1.2λ are plotted as curves 3 and 4 respectively. S-parameters for a 50Ω right-angled bend obtained by using the P-mesh code are listed in Table 3.12. Power radiated results for a 50Ω right-angled bend computed by using the P-Mesh code when the de-embedding section length is 0.7λ and 1.2λ are listed in Table 3.13.

Table 3.12: S-parameters for a right-angled bend obtained by using the P-Mesh Code.

<table>
<thead>
<tr>
<th>Frequency (GHz)</th>
<th>Length of De-embedding Section 0.7λ</th>
<th>Length of De-embedding Section 1.2λ</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>S11</td>
<td>S12</td>
</tr>
<tr>
<td>10</td>
<td>0.0679064</td>
<td>-86.82</td>
</tr>
<tr>
<td>20</td>
<td>0.1423098</td>
<td>-84.42</td>
</tr>
<tr>
<td>30</td>
<td>0.2284852</td>
<td>-84.48</td>
</tr>
<tr>
<td>40</td>
<td>0.329836</td>
<td>-85.91</td>
</tr>
</tbody>
</table>

Table 3.13: Power radiated results for a 50Ω right-angled bend.

<table>
<thead>
<tr>
<th>Frequency (GHz)</th>
<th>Length of De-embedding Section 0.7λ</th>
<th>Length of De-embedding Section 1.2λ</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>1.001298</td>
<td>-</td>
</tr>
<tr>
<td>20</td>
<td>1.004326</td>
<td>-</td>
</tr>
<tr>
<td>30</td>
<td>0.9829471</td>
<td>-17.68</td>
</tr>
<tr>
<td>40</td>
<td>0.9737684</td>
<td>-15.81</td>
</tr>
</tbody>
</table>
Results shown in Figure 3.20 indicate that power radiated results at 10 GHz using the P-Mesh code are in good agreement with PV results and to MNM results when the length of the de-embedding section is 1.2\lambda. However, at 30 GHz and 40 GHz the three sets of results are in good agreement when the length of the de-embedding section is 0.7\lambda.

Figure 3.19: Comparison of power radiated results from an open end, on \( \varepsilon_r = 2.2 \) substrate, with P-Mesh and PV Method.
Figure 3.20: Comparison of power radiated from a 50Ω bend, on \( \varepsilon_r = 2.2 \) substrate, with PV results and fullwave analysis results.
CHAPTER IV

EXPERIMENTS TO VERIFY RADIATION FROM MICROSTRIP BENDS

The multiport network model for evaluating radiation loss from microstrip discontinuities is verified experimentally by fabricating bent and linear resonators and comparing calculated and measured results of $Q$ factors of these resonators.

4.1 Design of Experiment

Since the actual radiated power from most microstrip discontinuities is too small to be measured accurately, the scheme that was explored for experimental verification of radiation loss from microstrip right-angled bends consists of four gap coupled resonant structures, shown in Figures 4.1(a-d). These resonators are short circuited at the two ends so that the radiation from the open ends does not dominate the total radiation loss. The structure in Figure 4.1(a) is approximately 2.5 wavelengths long and the bend is located at the voltage maximum. On the other hand, the circuit in Figure 4.1(b) is two wavelengths long, and a voltage null (or minimum) exists at the bend. The structures in Figures 4.1(c) and 4.1(d) are two wavelengths and 2.5 wavelengths-long straight transmission line resonators, respectively, included for experimental estimation of other losses. The transmission line impedance was selected to enhance the radiation from the bend discontinuity and, at the same time, to reduce the
power reflected from the discontinuity in order to detect the resonance easily from the measured values of $S_{11}$ and $S_{12}$. From radiation consideration a 20 ohm bend radiates more than a bend in a microstrip line with higher impedance but the computed results for $S_{21}$ for a 20Ω bend showed that the dip in $S_{12}$ cannot be detected since most of the power is reflected from the bend. Comparison of computed S-parameters for 20Ω, 25Ω, 30Ω, 35Ω and 50Ω indicated that the best choice is 35Ω resonators on $\epsilon_r = 2.2$, 0.02 inch thick. Duroid, $\epsilon_r = 2.2$, has been chosen so that radiation loss is enhanced and its measurement is simplified. In linear resonators (with the two ends shorted) the radiation losses are negligible. Thus the $Q$ measurement on these two resonators yields conductor and dielectric losses. Values of these types of losses are accounted for while calculating the radiation loss from the measured $Q$ factors for the bent resonators.

These resonators were printed on the same duroid substrate to minimize errors due to differences in dielectric constant and substrate thickness between different substrates.

### 4.2 Comparison of Measured and Calculated Resonant Frequencies

The resonant frequency of each resonator was calculated using the multiport network model for evaluating edge voltages along the resonator edges as described in Section 2.3.3. The configuration is broken down into elementary rectangular segments. The first and last segment are short circuit elements. The size of the $Z$-matrix of a short circuit element is $1 \times 1$ and the value of $Z_{11}$ has been taken as $10^{-6}$ (a sample check of edge voltages for different values of $Z_{11}$ between $10^{-5}$ to $10^{-7}$ showed that there is good convergence of edge voltages for $Z_{11}$ equal to $10^{-6}$). These resonators are short circuited at the two ends to eliminate radiation from the open ends. Theoretically, at resonance the electri-
Figure 4.1: Gap coupled resonant structures.
cal length of each resonator is a multiple of a half wavelength. The resonant frequency of each resonator was detected numerically by searching for voltage maximum at a quarter wavelength away from short circuit ends as a function of frequency. All measured results reported in this chapter were carried out using an HP 8510 automatic network analyzer.

A comparison of the measured and computed resonant frequencies is given in Table 4.1. The main reason for the differences between the calculated and measured resonance frequencies is the fact that the short circuits are not ideal and we have a small reactive (inductive) impedance present there.

Table 4.1: Comparison of measured and calculated resonant frequencies.

<table>
<thead>
<tr>
<th>Resonator Type</th>
<th>$f_r$ measured (GHz)</th>
<th>$f_r$ calculated (GHz)</th>
<th>$\Delta f$ (GHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2$\lambda$ linear</td>
<td>9.675</td>
<td>9.97000</td>
<td>0.2950</td>
</tr>
<tr>
<td>2$\lambda$ bent</td>
<td>9.910</td>
<td>10.15180</td>
<td>0.2418</td>
</tr>
<tr>
<td>2.5$\lambda$ linear</td>
<td>9.585</td>
<td>9.87085</td>
<td>0.2858</td>
</tr>
<tr>
<td>2.5$\lambda$ bent</td>
<td>9.635</td>
<td>9.83180</td>
<td>0.1968</td>
</tr>
</tbody>
</table>

The short circuit inductance is equivalent to an additional line length. By comparison of the experimental results with calculated values for 2$\lambda$ linear resonator, we find that the value of the short circuit inductance per unit line width is 14.57 pH/m. The value of the inductance based on measurements on the 2.5$\lambda$ line resonator is found to be 17.6 pH/m.

Calculated resonant frequencies given in Table 4.1 do not include these short circuit inductances.

One may note that the error in computation of the resonant frequency
caused by the short circuit inductance are two to three percent only. The short circuit inductance, $L_{eq}$, may be incorporated in the calculation by taking $Z_{11}$ equal to $j L_{eq}$.

4.3 Computation of $Q$ factors

There are three major types of losses in microstrip circuits: conductor losses, dielectric losses and radiation losses. Losses in a microstrip resonator may be expressed in terms of a $Q$ factor.

$$Q = \frac{2\pi f_r U_s}{P_l}$$  \hspace{1cm} (4.1)

where $U_s$ is the energy storage and $P_l$ is the average power lost in the resonator. The dielectric $Q$ factor $Q_d$ is inversely proportional to $\tan \delta$. The conductor loss $Q$ factor $Q_c$ is given by [21]

$$Q_c = h \sqrt{\mu_0 \pi f_r \sigma_c}$$  \hspace{1cm} (4.2)

where $h$ is the substrate height and $\sigma_c$ is the conductivity of the conductor. If we take surface roughness into account, $Q_c$ may be approximated as $Q_c^* = 0.625 Q_c$ [21]. The radiation $Q$ factor, $Q_r$, is given as

$$Q_r = \frac{2\pi f_r U_s}{P_r}$$  \hspace{1cm} (4.3)

where $P_r$ is the radiated power. In (4.1) and (4.3), the energy storage $U_s$ may be evaluated as

$$U_s = \frac{\epsilon_r \epsilon_0}{2h} \sum_{i=1}^{N} V^2(i) \ dx_i \ dy_i$$  \hspace{1cm} (4.4)

The resonator area is divided into $N$ cells and $V(i)$ is the voltage at the center of the $i$th unit cell. The area of the $i$th unit cell is represented by $dx_i dy_i$. Usually in the discontinuity region the voltage distribution varies considerably so there are more cells in the discontinuity region. The total $Q$ factor $Q$ may be written
as

\[ Q = \left( \frac{1}{Q_r} + \frac{1}{Q_c} + \frac{1}{Q_d} \right)^{-1} \quad (4.5) \]

Since the height of the substrate is less than 0.02 wavelength, surface wave losses are negligible. The calculated \( Q \) factors for various resonators are given in Table 4.2.

**Table 4.2: Calculated resonance frequency radiated power and \( Q \)-factors for the resonant structures.**

<table>
<thead>
<tr>
<th></th>
<th>2( \lambda ) bent</th>
<th>2.5( \lambda ) bent</th>
<th>2( \lambda ) linear</th>
<th>2.5( \lambda ) linear</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f_r ) GHz</td>
<td>10.386699</td>
<td>9.927564</td>
<td>10.000009225</td>
<td>10.000014</td>
</tr>
<tr>
<td>( P_r ) (Watts)*</td>
<td>180.885</td>
<td>175.68</td>
<td>123.98</td>
<td>72.61</td>
</tr>
<tr>
<td>( Q_r )</td>
<td>3036.08</td>
<td>1322.22</td>
<td>5884.82</td>
<td>6844.24</td>
</tr>
<tr>
<td>( Q_d )</td>
<td>1000</td>
<td>1000</td>
<td>1000</td>
<td>1000</td>
</tr>
<tr>
<td>( Q_c )</td>
<td>650.66</td>
<td>636.05</td>
<td>638.37</td>
<td>638.37</td>
</tr>
<tr>
<td>( Q )</td>
<td>348.88</td>
<td>300.34</td>
<td>365.44</td>
<td>368.65</td>
</tr>
</tbody>
</table>

(normalized for an input current of 1 amp)

In the calculation reported in this chapter, \( \tan \delta \) is equal to 0.01 and \( \sigma_c \) is equal to 4.8 \( 10^7 \). The resonance frequencies given in Table 4.2 were calculated as described in Section 4.2. \( Q_r \) was calculated using equations (4.3) and (4.4). \( Q_c \) was calculated using equation (4.2). The total \( Q \) factor \( Q \) was calculated using equation (4.5).

We note that in all four cases, conductor losses dominate. Radiation \( Q \)'s for resonators with bends is much smaller than those for corresponding linear resonators. If we ignore radiation from the two linear resonators, the total \( Q \) values \( \left( \frac{1}{Q_r} + \frac{1}{Q_c} \right)^{-1} \) are 389.63 in both cases (2\( \lambda \) and 2.5\( \lambda \) lengths). The bent resonator with 2\( \lambda \) length has a higher total \( Q \) (348.88) than the 2.5\( \lambda \) resonator (300.34) and this difference is caused by lower radiation in 2\( \lambda \) case (because of a voltage minimum at the bend).
4.4 Measurements of $Q$-factors

Since the calculated VSWR using circuit analysis based on MCAP computer program [38], and the measured VSWR values were around 4:1, we employed the transmission line method [10] to measure the $Q$ factor. In the transmission line method, the $S$ parameters $|S_{11}(f)|$ and $|S_{12}(f)|$ of the resonator are measured as a function of frequency, and the maximum value of $|S_{12}|$ or the minimum value of $|S_{11}|$ yields the resonant frequency $f_r$. The loaded $Q$ ($Q_L$) of the resonator is determined from the -3dB points of $|S_{12}(f)|$:

$$Q_L = \frac{f_r}{\Delta f}$$  \hspace{1cm} (4.6)

where $\Delta f$ is bandwidth at -3dB points. The unloaded $Q$ factor ($Q$) is obtained from [10]

$$Q = Q_L(1 + 2k)$$  \hspace{1cm} (4.7)

where $k$ is the coupling factor on both the source and load ports of the resonator. $k$ is evaluated as [10].

$$k(f_r) = \frac{|S_{11}(f_r)|}{2|1 - S_{12}(f_r)|}$$  \hspace{1cm} (4.8)

and unloaded $Q$ is obtained as [10]

$$Q(f_r) = \frac{Q_L(f_r)}{1 - |S_{12}(f_r)|}$$  \hspace{1cm} (4.9)

Measured resonance frequencies, values of $S_{11}$ and $S_{12}$ at resonance, and $\Delta f$ values for the -3dB bandwidth are listed in Table 4.3. Using equations 4.6 and 4.9, we calculate $Q_L$ and $Q$ values. $Q_L$ and $Q$ thus obtained are given in Table 4.4.
Table 4.3: Measured results for microstrip resonators.

<table>
<thead>
<tr>
<th>Resonator</th>
<th>f₀ (GHz)</th>
<th>S11 (dB)</th>
<th>S12 (dB)</th>
<th>Df (MHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2λ linear</td>
<td>9.675</td>
<td>-3.7</td>
<td>-16.71</td>
<td>57</td>
</tr>
<tr>
<td>2λ bent</td>
<td>9.91</td>
<td>-4.3</td>
<td>-13</td>
<td>69</td>
</tr>
<tr>
<td>2.5λ linear</td>
<td>9.585</td>
<td>-4.1</td>
<td>-25.77</td>
<td>51</td>
</tr>
<tr>
<td>2.5λ bent</td>
<td>9.635</td>
<td>-3.0</td>
<td>-17.89</td>
<td>70</td>
</tr>
</tbody>
</table>

Using Equations 21 and 23, we calculated Q_L and Q. Measured Q_L and Q are given in Table 6.

Table 4.4: Measured loaded and unloaded Q-factors

<table>
<thead>
<tr>
<th></th>
<th>Q_L</th>
<th>Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>2λ linear</td>
<td>169.73</td>
<td>198.75</td>
</tr>
<tr>
<td>2λ linear</td>
<td>143.62</td>
<td>185.04</td>
</tr>
<tr>
<td>2.5λ linear</td>
<td>187.94</td>
<td>198.25</td>
</tr>
<tr>
<td>2.5λ bent</td>
<td>137.64</td>
<td>157.75</td>
</tr>
</tbody>
</table>

4.5 Computation of Radiation Q-factors Accounting for Measured Losses in Linear Resonators

The losses in the 2λ and 2.5λ linear resonators are greater than the losses that were calculated theoretically because our calculations did not take surface roughness and correct values of conductor and dielectric loss parameters into account. This is demonstrated by measured values of Q in 2λ and 2.5λ linear resonators to be lower than the calculated values.

For experimental evaluation of the radiation Q associated with the bend resonators, we assume that the linear resonators have negligible radiation, and calculate other losses from measured Q-values. These losses are present for bent
resonators also and are subtracted from the total losses for bend resonators, in order to evaluate radiation from bends. The radiation $Q$ (thus calculated from measurements) is compared with the theoretical value based on multiport network model for radiation from discontinuities.

Thus, the experimental $Q_{\text{losses}}$ of the bent resonators may be written as

$$\frac{1}{Q_r} = \frac{1}{Q_{\text{measured}}} - \frac{1}{Q_{\text{losses}}}$$  \hspace{1cm} (4.10)

where $Q_{\text{losses}}$ is the measured $Q$ factor for $2\lambda$ and $2.5\lambda$ linear resonators, and $Q_{\text{measured}}$ is the measured $Q$ of bent resonators. Comparison between the measured and calculated radiation $Q$-factors is given in Table 4.5.

Table 4.5: Comparison of measured and calculated radiation $Q$-factors.

<table>
<thead>
<tr>
<th>$Q$ factor</th>
<th>$2\lambda$ bent resonators</th>
<th>$2.5\lambda$ bent resonator</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q_r$</td>
<td>Measured: 2682.47 Calculated: 2762.6</td>
<td>Measured: 772.19 Calculated: 876.95</td>
</tr>
<tr>
<td>$Q$</td>
<td>185.04</td>
<td>185.25</td>
</tr>
</tbody>
</table>

In the second row of Table 4.5, measured values are unloaded $Q$-values shown in Table 4.4, and the calculated values are based on radiation losses calculated from our model when the other losses (conductor plus dielectric, etc.) obtained from measurements on linear resonators.

The disagreement between the measured and calculated $Q_r$ for the $2\lambda$ bent resonator is 3% and for $2.5\lambda$ bent resonators, the corresponding value is 11%. The corresponding differences for total $Q$ are only 0.1% and 2.56%, respectively. Keeping in mind the experimental uncertainties and the assumption that the conductor losses are equal in the straight and bent resonators, the agreement
between theory and experiments is reasonable. This verifies the equivalent magnetic current modeling approach used for estimating radiation from microstrip discontinuities.
CHAPTER V

EVALUATION OF SPURIOUS COUPLING AMONG DISCONTINUITIES IN MICROSTRIP CIRCUITS

5.1 Approach

Radiation from microstrip discontinuities causes external electromagnetic coupling and generates undesired interaction among different parts of the circuit. This phenomena become significant in high density circuits, more bends and other discontinuities are introduced and spurious electromagnetic coupling increases considerably.

The spurious coupling between two discontinuities (due to external fields) may be incorporated in the multiport network model by connecting an additional multiport network between the two discontinuities as shown in Figure 5.1. The coupling network MCN is characterized in terms of an admittance matrix \([Y_m]\). Elements of this matrix represent mutual admittances between various sections of the edges of the two discontinuities. As described in Section 2.3 a planar multiport network model of the discontinuity configuration and the segmentation method are used to evaluate voltage distribution around the edges of the discontinuity configuration. This voltage distribution is expressed as an equivalent magnetic current line source distribution which is used to calculate the near zone fields at the location of the other discontinuity. Fields produced by a magnetic current element are given in Section 2.4, equations 2.12, 2.14
and 2.15. For evaluation of radiation loss from microstrip discontinuities the far zone radiation field is calculated and only the terms with \( \frac{1}{z} \) variation are considered. However, for evaluation of spurious coupling, we need to calculate fields at much shorter distances and terms in equations 2.14 and 2.15 with \( \frac{1}{z^2} \) and \( \frac{1}{z^3} \) can not be neglected.

![Network model for evaluating spurious coupling between discontinuities.](image)

**Figure 5.1:** Network model for evaluating spurious coupling between discontinuities.

### 5.2 Microstrip Edge Field and Associated Magnetic Current Distribution

The multiport network modeling approach is based on considering the upper surface of the microstrip circuit as a reference plane and evaluating the external fields in terms of equivalent sources on this reference plane. According to the Schelkunoff's equivalence theorem, we can postulate a conducting sheet located on this reference plane (considered a closed surface enclosing all of the sources) and locate the equivalent magnetic current sources in this plane. These equivalent magnetic current line sources can be placed at the physical edges
of the microstrip structure or at the edges of the configuration with effective
dimensions (obtained as a result of extending the boundary outward to account
for the fringing fields), as shown in Figure 2.2. Either of these approximations
is fairly good for calculation of the far-zone field. However, for evaluation of the
spurious coupling, we need to calculate fields at much shorter distances and it
is desirable to have a more accurate model for edge fields. The improved model
for edge fields, used for the present investigations, employs a two-dimensional
distribution of the equivalent magnetic current density in place of line current
sources used for radiation loss calculation. This allows us to account for the
variation of fringing field away from the edge. As shown in Figure 5.2 the
variation of the electric field at the dielectric-air interface outside of the edge
may be expressed as [39]

\[
E_{0X} (X \to 0^+) \approx V_0 \left( \frac{1 + \delta_e}{2\pi h} \right)^{1/2} \frac{1}{\sqrt{X}} \tag{5.1}
\]

\[
E_{0X} (X \to \infty) \approx \frac{V_0 h}{\pi} \left( \frac{1 - \delta_e}{1 + \delta_e} \right) \frac{1}{X^2} \tag{5.2}
\]

where \(X\) is the distance away from the edge, \(h\) is the thickness of the substrate,
and \(\delta_e\) is given by

\[
\delta_e = \frac{\varepsilon_r - 1}{\varepsilon_r + 1} \tag{5.3}
\]

\(V_0\) is found such that the integration of the magnetic current density along \(X\)
yields a value equal to the magnetic current line source described in Section 2.3.
Combining \(1/\sqrt{X}\) and \(1/X^2\) variations of (5.1) and (5.2) respectively, the variation
of the electric fringing fields may be written as

\[
E_{0X} = \frac{1}{aX^2 + b\sqrt{X}} \tag{5.4}
\]

where
Figure 5.2: Edge voltage distribution.
\[ b = \frac{1}{V_0 \left( \frac{1 + \xi_s}{2} \right)^{1/2}} \]

and

\[ a = \frac{1}{V_0 h \frac{1 + \xi_s}{1 + 2\xi_s}} \]

\( V_0 \) is given by

\[ V_0 = \frac{V_i}{\int_{X_0}^{X_1} \frac{1}{aX^2 + b} dX} \quad (5.5) \]

\( V_i \) is the voltage at the \( i \)-th section computed by using two-dimensional planar analysis as described in Section 2.3. \( X_0 \) is the width of the two-dimensional distribution of the equivalent magnetic current density.

The equivalent magnetic current density is given by \( -(n \times E_{0X}) \) and may be written as

\[ M(x) = \left| E_{0X}(X, z = h) \right| \quad (5.6) \]

### 5.3 Interaction Between Magnetic Currents and Mutual Coupling

The terms of the matrix \([Y_m]\), representing the MCN of Figure 5.1 may be obtained by representing the fringing field at the microstrip edges by equivalent magnetic line sources or by representing the fringing fields by small two-dimensional strips (length \( dt \) and width \( X_0 \)) with magnetic current distribution along \( X \) being given in equation 5.4. Each section of length \( dt \) is considered a summation of linear sources (each of length \( dt \)) distributed over a width \( X_0 \) touching the physical edge of the microstrip structure.
5.3.1 Linear Distribution of Magnetic Current

The mutual admittance terms may be evaluated by using reciprocity and reaction theory when linear sources are involved. The reciprocity theorem states that a response of a system to a source is unchanged when source and measurer are interchanged. Consider two sets of a–c sources \( J^a, M^a \) and \( J^b, M^b \) of the same frequency, existing in the same linear medium. The fields produced by the a sources are \( E^a \) and \( H^a \) and the fields produced by the b sources are \( E^b, H^b \). Lorenz reciprocity theorem may be written as [40].

\[
- \iiint (E^a \times H^b - E^b \times H^a) \cdot ds = \iiint (E^a \cdot J^b - H^a \cdot M^b - E^b \cdot J^a + H^b \cdot M^a) dV
\]  

(5.7)

The left hand term of equation (5.7), integrated over a sphere of radius \( r \to \infty \) is equal to zero. Equation (5.7) reduces to

\[
\iiint_V (E^a \cdot J^b - H^a \cdot M^b) dV = \iiint_V (E^b \cdot J^a - H^b \cdot M^a) dV
\]

(5.8)

Equation (5.8) is called the reaction theorem. The reaction theorem state that the reaction of field \( a \) on source \( b \) is equal to the reaction of field \( b \) on source \( a \). For linear magnetic current source we can write equation (5.8) when \( J^b = 0 \) as

\[
- \oint H^a \cdot K^b d\ell = -K^b \oint H^a \cdot d\ell = V^b I^a
\]

(5.9)

where \( K^b \) is the magnetic currents in volts, \( K^b = -V^b \), and \( I^a \) is the current at the location of the b source due to some source \( (K^a) \). To relate the electromagnetic reciprocity theorem to reciprocity theorem in circuit theory we may consider a two-port network as shown in Figure 5.3. The characteristics of a linear network can be described by the admittance matrix–\([Y]\)

\[
\begin{bmatrix}
I_1 \\
I_2
\end{bmatrix} = \begin{bmatrix}
Y_{11} & Y_{12} \\
Y_{21} & Y_{22}
\end{bmatrix} \begin{bmatrix}
V_1 \\
V_2
\end{bmatrix}
\]

(5.10)
The partial response $I_{ij}$ is the current at port $i$ due to source $V_j$ at port $j$, when other ports are short-circuited (i.e. other voltages are zero). Hence

$$Y_{ij} = \frac{I_{ij}}{V_j} \quad (5.11)$$

The terms of matrix $[Y_m]$, representing the MCN of Figure 5.1 are obtained by using equation (5.11). The edge fields are modeled by small sections of length $dl$ of equivalent magnetic current line source. The current induced in the j-th element is calculated from the magnetic field at that location as

$$J_j = (\hat{z} \times \vec{H}) \cdot \hat{j} \quad (5.12)$$

where $\hat{z}$ is a unit vector in the Z-direction and $\hat{j}$ is a unit vector normal to the j-th element. $\vec{H}$ is the magnetic field ($H_\theta$, $H_r$) produced at the j-th subsection of the nearby discontinuity and is given in equations (2.14) and (2.15) as

$$H_\theta = \frac{j k_0 M dl \sin \theta}{4 \pi \eta_0 r} \left(1 + \frac{1}{j k_0 r} - \frac{1}{(k_0 r)^2} \right) e^{-j k_0 r} \quad (5.13)$$

$$H_r = \frac{M dl \cos \theta}{2 \pi \eta_0 r^2} \left(1 + \frac{1}{j k_0 r} \right) e^{-j k_0 r} \quad (5.14)$$

The value of $Y_{ij}$ is obtained as the ratio of the current induced in the j-th subsection to the voltage $V_i$ (producing this current) at the i-th subsection.

$$Y_{ij} = J_j dl_j / V_i \quad (5.15)$$

5.3.2 Two-Dimensional Distribution of Magnetic Current

When a two-dimensional distribution of the equivalent magnetic current density is used in place of line current sources, elements of matrix $[Y_m]$ may be evaluated from computation of power flow from and in between the two sources. The power flow is evaluated by using Poynting vector theorem related to the fields produced by sources $M$ and $J$. The vector identity
\[ \nabla \cdot (E \times H^*) = H^* \cdot \nabla \times E - E \cdot \nabla \times H^* \] (5.16)

is used to evaluate the Poynting vector \(E \times H^*\). Using Maxwell's equations

\[ \nabla \times H = \frac{\partial D}{\partial t} + J^c + J^i = J^t \] (5.17)
\[ \nabla \times E = -(\frac{\partial B}{\partial t} + M^i) = -M^t \] (5.18)

where superscripts \(t, c, i\) denote total, conduction and impressed currents. A scalar multiplication of equation (5.17) by \(E\) yields

\[ E \cdot (\nabla \times H^*) = E \cdot J^t \]

A scalar multiplication of equation (5.18) by \(H^*\) yields

\[ H^* \cdot (\nabla \times E) = -H^* \cdot M^t \]

Substituting equations (5.17) and (5.18) in (5.16) we can write equation (5.16) as

\[ \nabla \cdot (E \times H^*) = -H^* \cdot M^t - E \cdot J^t \] (5.19)
Integrating equation (5.19) throughout a region and applying the divergence theorem the result may be written as

\[ \int \int E \times H^* \cdot ds = -\int_V E \cdot J^* + H^* \cdot M^i dV \]  

(5.20)

\[ E \cdot J^i = \frac{\partial}{\partial t} \left( \frac{\varepsilon E^2}{2} \right) + \sigma E^2 + E \cdot J^i \]  

(5.21)

\[ H \cdot M^i = \frac{\partial}{\partial t} \left( \frac{\mu H^2}{2} \right) + H \cdot M^i \]

The net electric and magnetic energies within a region are defined as

\[ W_e = \frac{1}{2} \int_V \varepsilon E^2 dV \]  

(5.22)

\[ W_m = \frac{1}{2} \int_V \mu H^2 dV \]

The net power converted to heat energy is defined as

\[ P_d = \int_V \sigma E^2 dV \]  

(5.23)

The net power supplied by sources within the region is defined as

\[ P_{\text{flow}} = -\int_V (E \cdot J^i + H \cdot M^i) dV \]  

(5.24)

The average power flow due to the impressed electric and magnetic currents \( J \) and \( M \) may be written as

\[ P_{\text{flow}} = -\frac{1}{2} \int_s (H^* \cdot M + E \cdot J^*) dS \]  

(5.25)

where the surface \( S \) encloses both sources and \( E \) and \( H \) are the fields on \( S \) caused by both sources. For \( J \) equal to zero the power flow may be written as

\[ P_{\text{flow}} = -\frac{1}{2} \int_s H^* \cdot M ds = -\frac{1}{2} \int_s (H_1 + H_2)^* \cdot (M_1 + M_2) dS \]  

(5.26)
On the other hand the power flow in the two port network shown in Figure 5.3 may be written as

\[ P_{\text{network}} = \frac{1}{2} (V_1^* V_2^* \cdot \left( \begin{array}{c} I_1 \\ I_2 \end{array} \right) = \frac{1}{2} (V_1^* Y_{11} V_1 + V_1^* Y_{12} V_2 + V_2^* Y_{21} V_1 + V_2^* Y_{22} V_2) \]  

(5.27)

Comparing equations (5.26) and (5.27) and after identification of the power contributions from each aperture the mutual admittance may be written as

\[ Y_{12}^* = -\frac{1}{V_1^* V_2} \int \int (H_1^* \cdot M_2) \cdot dS_2 \]  

(5.28)

where \( H_1 \) is the magnetic field produced by the source \( M_1 \) at the location of the source \( M_2 \), \( H_1 \) may be written as

\[ H_1 = \int M_1 \cdot \vec{H}_1 \ dS_1 \]  

(5.29)

where \( \vec{H} \) is a 3 x 3 matrix containing the field components of the \( \vec{H} \) field at the second source caused by the three components of the infinitesimal current element at location 1. The elements of the matrix \( \vec{H} \) are found from the fields of an infinitesimally small magnetic dipole as described in Section 2.4 and are given in equations (5.13) and (5.14). We obtain a four-dimensional integral for the mutual admittance:

\[ Y_{12} = -\frac{1}{V_1^* V_2} \int_{S_1} \int_{S_2} M_1^* (r_1) \cdot \vec{H} (r_2 - r_1) \cdot M_2 (r_2) dS_2 dS_1 \]  

(5.30)

where

\[ r = (\Delta x^2 + \Delta y^2 + \Delta z^2)^{1/2} = |r_2 - r_1| \]
The terms of the matrix \( [\mathbf{Y}_m] \) representing the MCN of Figure 5.1 are obtained by representing the edge fields by small two-dimensional strips (length \( d\ell \) and width \( X_0 \) with magnetic current distribution along \( X \) being as in equation (5.4). When the length of each port \( d\ell \) is very small the voltage distribution along \( d\ell \) is assumed to be uniform. The four-dimensional integral reduces to two-dimensional integral

\[
Y_{ij} = -\frac{d\ell_i d\ell_j}{V_i V_j} \int \int M_i^*(r_i) \cdot \vec{H}(r_j - r_i) \cdot H_j(r_j) dr_j dr_i
\]  \hspace{1cm} \text{(5.31)}

Where \( d\ell_i \) is the length of the \( i \)-th port. Each section of length \( d\ell \) can be considered a summation of linear sources (each of length \( d\ell \)) distributed over a width \( \Delta X \) touching the physical edge of the microstrip discontinuity.

When the distribution width \( \Delta X \) is smaller than the distance between the two discontinuities, the double integration in equation 5.31 may be approximated as a single integral over \( dr_i \). If

\[
r_3 - r_1 >> \Delta X
\]

(in Figure 5.4) then

\[
\vec{H}(r_3 - r_1) \approx \vec{H}(r_3 - r_2)
\]

\[
Y_{ij} = -\frac{d\ell_i d\ell_j}{V_i V_j} \int M_i^*(r_i) \cdot \vec{H}(r_3 - r_i) dr_i \int M_j(r_j) dr_j
\]  \hspace{1cm} \text{(5.32)}

where

\[
V_j = \int M_j(r_j) dr_j
\]

and
\[ Y_{ij} \approx -\frac{d\ell_i d\ell_j}{V_i} \int M_i^*(r_i) \cdot \bar{H}(r_b - r_i) dr_i \] (5.33)

![Diagram of coupling between two apertures](image)

Figure 5.4: Coupling between two apertures.

To summarize, we have presented three different approximations to evaluate \([Y_{ij}]\). In the first approximation the fringing field at the microstrip edges are modeled by equivalent magnetic line sources. This approximation is fairly good for calculations of the far-zone field. However, for evaluation of spurious coupling, we need to calculate fields at much shorter distances and it is desirable to have a more accurate model for edge fields. The second approximation (and most rigorous out of the three discussed) employs a two-dimensional distribution of the equivalent magnetic current density around the edges of both discontinuities. When the distribution width is much smaller than the distance between the two discontinuities the double integration in equation (5.31) may be reduced to single integration as given in equation (5.33). This simplification yields the third approximation described in this section.
Diagonal terms $Y_n$ in the admittance matrix $[Y_m]$ are computed by evaluating the radiation conductance associated with the $i$-th subsection of the microstrip edge as described in Section 2.3. The $Z$-matrix of the mutual coupling network is the inverse of the matrix $[Y_m]$. The segmentation method is used to combine the $Z$-matrix representations of the discontinuities and that of the coupling network to yield the overall $Z$-matrix. The resulting $Z$-matrix is converted to $S$-parameters. The effect of the coupling on the circuit performance is obtained from these $S$-parameters.

5.4 Coupling Calculations

In Sections 5.1 to 5.3, the approach used to evaluate spurious coupling among discontinuities in microstrip circuits has been described in detail. In this section, the sequence of major steps needed for calculation of spurious coupling between two discontinuities is listed. These are:

Steps in Evaluation of External Coupling Between Two Discontinuities $A$ and $B$ in a MMIC

1. Divide component $A$ into regular segments and calculate $Z_A$ matrix, as described in Section 2.3.

2. Divide component $B$ into regular segments and calculate $Z_B$ matrix, as described in Section 2.3.

3. Represent the edge fields by small sections of strips length $dl$ and width $X_0$, as shown in Figure 5.5. At $X_0$ equal to twice the substrate height the field is less than 0.1 of the value at the edge as shown in Figure 5.2. $S_{13}$ results shown in Figure 7.3 for coupling between two corners of right-angled bends for different values of $X_0$ indicate that for spacing values greater than ten times the height of the substrate there are minor changes in $S_{13}$ for values of $X_0$ greater than twice the substrate height.
4. Calculate the magnetic field \( H_{\theta} \), \( H_{r} \) (\( H_{\theta} \) given in equation (5.13), \( H_{r} \) given in equation (5.14)) at the j-th subsection of component B excited by the i-th element of magnetic current at component A.

5. Calculate the \( Y_{ii} \) element of the mutual coupling admittance matrix as in equation 2.6

6. Calculate the \( Y_{ij} \) element of the mutual coupling admittance matrix, as in equation (5.31).

7. Employ the same procedure to compute all the elements of the coupling matrix \([Y_m]\).

8. Invert the matrix \([Y_m]\) to obtain matrix \([Z_C]\).

9. Use segmentation method to combine \( Z_A \) with \( Z_C \) to produce \( Z_{AC} \).

10. Use segmentation method to combine matrix \( Z_{AC} \) with \( Z_B \) to produce \( Z \)-matrix representation of the coupled elements.

11. Transform \( Z \)-matrix to S-parameter to yield coupling between component A and B.

![Figure 5.5: Equivalent magnetic current distribution.](image-url)
5.5 Verification of Coupling Calculation

Since the results for coupling between two parallel lines are well known, this configuration has been used for verifying the model proposed in this chapter. Extensive computations have been carried out for coupling at 10 GHz between two quarter-wave long 50Ω (in absence of coupling) lines located parallel to each other on a substrate with εr = 2.2 and a thickness of 0.01 inch. Each edge of the two microstrip lines is divided into 5 sections. The size of the Z-matrix for each microstrip line section is (12 x 12) and the size of the Z-matrix for the mutual coupling network becomes (20 x 20). The Z-matrices for the two microstrip lines are combined with the mutual coupling matrix to yield the (4 x 4) Z-matrix with reference to the four ports located at the ends of the two parallel lines. The S-matrix of the coupled line network has been compared to the quasistatic computations based on the Bryant and Weiss approach [41]. This comparison is shown in Figure 5.6. We notice a fairly good agreement which verifies the coupling model developed in this research. The coupling parameter S13 (in dB) is plotted in Figure 5.6. S13 results using quasistatic analysis are plotted as curve 1. S13 results using MNM (approximation number 2, equation (5.31), in Section 5.3) are plotted as curve 2. S13 results using MNM (approximation number 3, equation (5.33), in Section 5.3) are plotted as curve 3. In both of these cases, the limit X0 (width of the magnetic current sheet) is determined by checking convergence of coupling results for different values of X0. The differences between the three sets of results is summarized in Table 5.1. From Table 5.1 it is clear that MNM using equivalent magnetic current line source should not be used for evaluation of spurious coupling between two parallel lines when the spacing between the discontinuities is less than twenty-four times the substrate thickness. However, MNM using a
two-dimensional distribution of the equivalent magnetic current density can be applied to evaluation of spurious coupling between parallel strips. Also single integration simplification (approximation number 3) yields results very close to more rigorous approximation number 2.

Table 5.1: Comparison of coupling $S_{13}$ (in dB) computed by MNM approach with quasistatic results for two parallel strips.

<table>
<thead>
<tr>
<th>Spacing (1 mm)</th>
<th>Spacing (in terms of h)</th>
<th>Quasistatic Computation</th>
<th>MNM Line Source</th>
<th>Approach Distribution At Both Strips</th>
<th>Approach Distribution At One Strip</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>7.87</td>
<td>-39.5</td>
<td>-53.19</td>
<td>-39.00</td>
<td>-39.60</td>
</tr>
<tr>
<td>4</td>
<td>15.75</td>
<td>-49.3</td>
<td>-68.22</td>
<td>-49.82</td>
<td>-49.40</td>
</tr>
<tr>
<td>6</td>
<td>23.62</td>
<td>-55.5</td>
<td>-83.22</td>
<td>-55.25</td>
<td>-55.50</td>
</tr>
</tbody>
</table>

The spacing values listed in this table correspond to $d/h$ varying from 8 through 24. We notice that the differences between three sets of results increases with spacing. The quasistatic results are based on electrostatic (capacitive) and magnetic coupling between the strips. Dynamic coupling caused by radiation is not included in the quasistatic computations. At large distances capacitive and magnetic coupling decreases, and contribution of the radiative coupling becomes more significant. This accounts for slightly larger values of coupling obtained by MNM method when the spacing between two line sections is increased.
Figure 5.6: Comparison of $S_{13}$ for parallel coupled lines.
CHAPTER VI

EXPERIMENTAL VERIFICATIONS OF THE MULTIPORT NETWORK MODEL FOR EVALUATING SPURIOUS COUPLING AMONG DISCONTINUITIES IN MICROSTRIP CIRCUITS

6.1 Two Configurations Used for Experimental Verification

Two different experimental verifications of the proposed model for evaluating spurious coupling among discontinuities in microstrip circuits are described in this chapter.

The first experimental verification was obtained by comparing the proposed method with the experimental results on a double-stub circuit obtained from Texas Instruments [42]. The second experimental verification was obtained by comparing computed results, using the proposed model, for coupling at 10 GHz between two parallel lines with step discontinuity from $50\Omega$ to $20\Omega$ with measured results obtained by using an HP 8510 Automatic Network Analyzer.

6.2 Interaction Between Two Open-Ended Stubs

6.2.1 Configuration

In this section the interaction between two open-ended $50\Omega$ microstrip stubs located on opposite sides of a $50\Omega$ microstrip line is investigated. The stubs lengths are quarter wavelength at approximately 10 GHz.

The experimental configuration shown in Figure 6.1 was fabricated on
5 mil alumina substrate with $\varepsilon_r = 9.9$[42].

This circuit has been analyzed using the Multiport Network Model with and without taking coupling effects into account. The numerical results are discussed in Section 6.2. In Subsection 6.2.4 the MNM results are compared to measured results and to fullwave analysis results obtained by using the BCW Code and the P-Mesh Code [42, 19 and 18].

6.2.2 Voltage Distribution Around the Edges of the Double Stub Circuit

The multiport network model is used to evaluate voltage distribution around the edges of the circuit as described in Section 2.3. Voltage levels at the various edges in the double stub circuit when excited by a unit (1 Amp) current (50Ω source impedance) at the input port with the other port terminated on 50Ω load are shown in Figure 6.2(a). This distribution is calculated at 9.825 GHz where the stub lengths are approximately quarter wave long.

As expected, the maximum voltage (48.8 Volts) appears at the open end of the stub nearer to the input end and decreases monotonically to small values at the junction of the stub and the main line since the first stub presents a virtual short at the main line, most of the power is reflected back and the voltage levels on the second stub are much smaller. Phase values for these voltage levels are shown in Figure 6.2(b). One may note that the phase distribution of the edge voltages is asymmetric near the discontinuity junction. Since the first stub presents a virtual short circuit at the main line, the voltage distribution along the main line between the two stubs is out of phase (at opposite ports).

6.2.3 Effect of Radiation Loss on Transmission Coefficient

The multiport network model is used to analyze the double-stub cir-
Figure 6.1: Configuration of the double-stub circuit.
Figure 6.2(a): Edge voltages (in volts) at various points along the double-stub circuit.
Figure 6.2(b): Phase distribution of the edge voltages for the double-stub circuit.
cuit without taking radiation and coupling effects into account. When the
two-dimensional planar analysis of the circuit is carried out, the higher order
evanescent modes are considered in the microstrip line section connecting the
two T-junctions. The results obtained are plotted as curve (1) in Figure 6.3.
We can see slight splitting of the dip into two dips in this response even when
no external interaction or radiation is included. Using the procedure described
in Section 2.7, S-parameters of the double-stub circuit were computed taking
radiation effects into account but without taking parasitic coupling interaction
into account. The power radiated at 10 GHz from the double stub circuit is
computed to be -20 dB.

Transmission coefficient with radiation included is plotted as curve 2 in
Figure 6.3. One may note that the effect of radiation loss on $S_{12}$ is negligible.
Detailed comparison of various results near the dip in the transmission coeffi-
cient $S_{12}$ are shown in Figure 6.3. Also included in this figure are the results for
the case when the two T-junctions are considered isolated from each other and
no higher mode (internal) interaction or parasitic coupling interaction is taken
into account. This is shown as curve 3 with single dip exhibiting no splitting
at all.

One may conclude that a slight split in the dip is generated due to
excitation of higher-order modes by the T-junction discontinuity.

6.2.4 Effect of Parasitic Coupling on S-parameters

The spurious coupling between the two stubs and the two T- junctions is
incorporated in the multiport network model by connecting a mutual coupling
network between the two stubs and the two T-junctions, as described in Chapter
5. The MCN for the double-stub circuit is shown in Figure 6.4. The ports' width
Figure 6.3: $S_{21}$ results near resonance of the two stubs. Curve 3 considers isolated discontinuities, while curve 1 accounts for internal interaction.
along the stub edges are $\frac{\lambda}{2}$ at 10 GHz. The total number of ports along the circuit edges is 26. Transmission results are not affected considerably when the number of ports is increased from 3 to 5 along the stub edges and along the open-end edges. Results for the transmission coefficient are plotted in Figure 6.5 for frequency ranges from 2.5 GHz to 26 GHz. Multiport network model results, curve 1, are compared with measured values [42], curve 2, and fullwave numerical analysis results [19], BCW Code, curve 3. Agreement between the MNM results and measured values is of the same order as that between the measured results and the fullwave results. One may note that the P-Mesh Code results [18] for $S_{12}$ are approximately the average between fullwave results, BCW Code, and MNM results as shown in Figure 6.6. In the BCW Code results, losses are neglected. However, in the MNM results, losses are included. Results for the reflection coefficient $S_{11}$ in dB are plotted in Figure 6.7 for frequency ranges from 2.5 GHz to 26 GHz. Multiport network model results, curve 1, are compared with measured values, curve 2, and fullwave numerical results, curve 3. There is a very good agreement between measured results for $S_{11}$ and MNM results.

The difference between these three sets of results is that of minor discrepancies in frequencies where values of $S_{11}$ and $S_{12}$ become very small. These dips correspond to the resonances of two stubs with effective length $\lambda/4$, $\lambda/2$ etc. It may be recalled that these resonance frequencies depend critically on the fringing capacitance at open ends of the two stubs. For MNM calculations, the open-end capacitance values are based on Hammerstal's results [43].

The MNM modeling approach allows one to take into account selective partial couplings also. Partial coupling results between different parts of the circuit are shown in Figure 6.8. $S_{12}$ results when spurious coupling between
Figure 6.4: MCN for the double-stub circuit.
Figure 6.6: Comparison of $S_{12}$ (in dBs) obtained from MNM approach with P-Mesh Code.

sections 2 and 3 is taken into account are plotted as curve 1. $S_{12}$ results when spurious coupling between sections (1 and 2) and (1' and 2') is taken into account are plotted as curve 2. One may note that a slight split in the dip is generated due to spurious coupling between these two sections. $S_{12}$ results when spurious coupling between sections 1 and 1' is taken into account are plotted as curve 3. $S_{12}$ results when spurious coupling between sections 2 and 2' is taken into account are plotted as curve 4. One may note that also in this case a slight split in the dip is generated due to spurious coupling between sections 2 and 2'. One may conclude that higher coupling values are obtained due to coupling between ports on the microstrip line and ports on the stub. However, spurious coupling between ports along the first T-junction and ports along the second
Figure 6.7: Comparison of MNNM approach results for $S_{11}$ (in dBs) with fullwave results and measured results.
T-junction is causing a slight split in the dip.

We may conclude that a splitting of the dip in $S_{12}$ response of the double-stub circuit is caused by interaction among higher-order modes along the microstrip section between the two stubs. Furthermore, this splitting of the dip is enhanced considerably by external parasitic coupling between the two stubs. This circuit is a good example wherein the parasitic coupling needs to be taken into account for accurate performance characterization.

6.3 Coupling Between Two Parallel Lines with Step Discontinuity and Comparison to Measured Results

Two parallel lines with step discontinuity are shown in Figure 6.9. Extensive computations have been carried out for coupling at 10 GHz between two parallel lines with step discontinuity from 50Ω to 20Ω (and back to 50Ω as shown in Figure 6.9. The 20Ω microstrip line is half a wavelength at 10 GHz on a substrate with $\varepsilon_r = 2.2$ and thickness of 0.02 inch. Two circuits with spacing of 1.5 mm and 2 mm between the parallel 20Ω lines were fabricated on a duroid substrate.

For computation of spurious coupling as described in Section 5.4, the configuration is broken down into three coupled line sections, 50Ω coupled line section and a 20Ω coupled line section, the third section being a 50Ω coupled line section. A multiport network representation of the coupling network is shown in Figure 6.10.

Comparison between measured and calculated results is given in Table 6.1. These values include coupling across the gap, as well as coupling among the step discontinuity. The differences in $S_{13}$ between calculated and measured results are 0.57 dB for spacing of 1.5 mm (≈ 3h) and a 0.61 dB for spacing of 2 mm (≈ 4h). The discrepancy between calculated and measured results for $S_{14}$ are
Figure 6.8: Partial coupling between different parts of the double-stub circuit.
Figure 6.9: Two parallel lines with step discontinuity.

1.9 dB for 1.5 mm spacing and 0.386 dB for a 2 mm spacing. The major reason for the differences between measured and calculated results is additional losses in the circuit due to surface roughness and conductor losses that were not taken into account in the calculation. In fact the measured losses for 1.5 mm spacing are greater by ~ 1 dB from the calculated losses, as noted by comparison of $S_{12}$ values.

Table 6.1: Comparison between calculated and measured results at 10 GHz of coupling between two parallel lines with step discontinuity.

<table>
<thead>
<tr>
<th>Measured Results</th>
<th>Calculated Results</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>d=1.5mm d=2mm</td>
</tr>
<tr>
<td></td>
<td>d=1.5mm d=2mm</td>
</tr>
<tr>
<td>S11</td>
<td>-17.09 -28.70</td>
</tr>
<tr>
<td></td>
<td>-9.80 -9.70</td>
</tr>
<tr>
<td>S12</td>
<td>-1.66 -1.24</td>
</tr>
<tr>
<td></td>
<td>-0.65 -0.65</td>
</tr>
<tr>
<td>S13</td>
<td>-21.58 -22.68</td>
</tr>
<tr>
<td></td>
<td>-21.01 -23.39</td>
</tr>
<tr>
<td>S14</td>
<td>-25.60 -26.14</td>
</tr>
<tr>
<td></td>
<td>-23.70 -25.75</td>
</tr>
</tbody>
</table>

One may note that there is a good agreement between measured and cal-
culated $S_{13}$ and $S_{14}$ which verifies the proposed model for estimation of coupling between discontinuities in microstrip circuits.

Figure 6.10: Multiport network representation of coupling between two circuit components.
CHAPTER VII

MNM RESULTS FOR SPURIOUS COUPLING

This chapter includes several examples where the multiport network model has been applied to evaluate parasitic coupling. Configurations investigated are: two-right-angled bends, corners of two right-angled bends, a gap in a microstrip line and corners of three right-angled bends. MNM results are compared with fullwave analysis results.

7.1 Computational Details for MNM Approach for Evaluation of Spurious Coupling

Computational details for evaluation of edge voltages and the Z-matrix of the discontinuity were given in Chapter 3. Steps in evaluation of external coupling between two discontinuities were given in Section 5.4. In coupling calculation we represent the edge fields by small sections of strips with length \( dt \) and width \( X_0 \). To improve computation accuracy and to minimize computation time, several computations were performed to determine the distribution width \( X_0 \). Coupling results, \( S_{14} \), in dB at 10 GHz between two corners of right-angled bends, on a substrate with \( \varepsilon_r = 2.2 \) 10 mil thick, are given in Table 7.1 for different values of \( X_0 \). \( S_{14} \) results given in Table 7.1 were computed using approximation number two in Section 5.3, equation 5.31. The differences in \( S_{14} \) results for \( X_0 \) equal to twice the substrate thickness and for \( X_0 \) equal to three times the substrate thickness, for spacing values greater than three millimeters,
are less than 0.42 dB. However, for spacing of two millimeters \( S_{14} \) results diverge as the distribution width \( X_0 \) increases from one time to three times the substrate thickness. The reasoning to this phenomena is that for spacing values less than eight times the substrate thickness as we increase the distribution width the distance \( |r_j - r_i| \) in equation (5.31) decreases and the magnetic field \( H(r_j - r_i) \) increase considerably. Since the magnetic field \( H(r_j - r_i) \) varies as \( \frac{1}{|r_j - r_i|} \) in the near-zone, as given in equations (5.13) and (5.14). One may conclude that coupling results using approximation number two in Section 5.3 will converge as long as the distance between the discontinuities is greater than four times the distribution width. At \( X_0 \) equal to twice the substrate height the field is less than 0.1 of the value at the edge, as shown in Figure 5.2. So, one may conclude that coupling results using the MNM approach may be accurate only for spacing values greater than eight times the thickness of the substrate. However, for spacing values less than 8\( h \) we can obtain a good estimate of the coupling results using approximation number two (or approximation number three) with distribution width equal to the substrate height.

<table>
<thead>
<tr>
<th>Spacing (in mm)</th>
<th>( X_0=h )</th>
<th>( S_{14} ) (in dB) ( X_0=2h )</th>
<th>( X_0=3h )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2(-8h)</td>
<td>-44.5</td>
<td>-42.41</td>
<td>-38.68</td>
</tr>
<tr>
<td>3(-12h)</td>
<td>-52.85</td>
<td>-51.91</td>
<td>-50.71</td>
</tr>
<tr>
<td>4(-16h)</td>
<td>-58.71</td>
<td>-58.08</td>
<td>-57.52</td>
</tr>
<tr>
<td>5(-20h)</td>
<td>-63.12</td>
<td>-62.61</td>
<td>-62.28</td>
</tr>
</tbody>
</table>

Table 7.1: Coupling results \( S_{14} \), between two corners of right-angled bends for different values of the distribution width.
7.2 Coupling Between Two Right-Angled Bends

Computed results for coupling between two right-angled bends are shown in Figures 7.1 and 7.2. In both cases, the line impedances are 50Ω, on a substrate with $\varepsilon_r = 2.2$, 10 mil thick, and the length of the coupled line section is $0.42\lambda$. Results in Figure 7.1 present the values of spurious coupling ($S_{13}$) when interactions only among the fields at the outer corners are taken into account. Each edge of the corner is divided into four sections, and the resulting MCN matrix is $(16 \times 16)$ in size. Coupling values have been computed for three different frequencies and are plotted as a function of distance between the two corners (normalized with respect to the substrate thickness). Coupling value at 40 GHz is -45.42 dB for $d/h = 8$. These coupling values increase considerably when the coupling between the parallel sections of microstrip lines is also taken into account. Results of this case are shown in Figure 7.2. For $d/h = 8$ the coupling value at 10 GHz is -39.7 dB. For $d/h = 16$, the coupling value at 10 GHz is -53.67 dB. Results in Figure 7.3 present the values of spurious coupling ($S_{13}$) at 10 GHz when interactions only among the fields at the outer corners are taken into account, for three different magnetic current distribution widths ranging from one to three times the substrate height.

The differences in $S_{13}$ results for 3 mm spacing between the corners when the distribution width $X_0$ is equal to twice the substrate height and to three times the substrate height is 1.19 dB and for 5 mm spacing the differences in $S_{13}$ results are 0.67 dB. One may note that for larger distances between the corners, $S_{13}$ values converge faster as a function of the distribution width.

Comparisons of coupling values $S_{13}$ between two corners of right-angled bends at 40 GHz are shown in Figure 7.4. $S_{13}$ results are computed using the MNM with distributed sources along the edges of the two corners and are
Figure 7.1: Coupling between corner of a right-angled bend as a function of spacing $d$ (in mm).
Figure 7.2: $S_{13}$ results between 50Ω bends on substrate with
$\varepsilon_r = 2.2$ 10 mil thick. $L = 1.055\,\text{cm}$.
Figure 7.3: Convergence of $S_{13}$ as a function of the distribution width.
plotted as curve 1. $S_{13}$ results using the MNM with distributed source along the edges of only one corner are plotted as curve 2. One may note that at distances greater than 3mm ($d/h \approx 12$) between the corners there is a good agreement ($\sim 1dB$) between these two sets of results.

Computation results for coupling ($S_{13}$) between two right-angled bends when the length of the coupled section is a quarter wavelength are plotted in Figure 7.5 for three different frequencies. The line impedance is 50Ω and the substrate has $\epsilon_r = 2.2$ and 10 mil thick. For $d/h = 8$ the coupling value $S_{13}$ at 10 GHz is -40.1 dB. One may note by comparison of $S_{13}$ results shown in Figure 7.2 and $S_{13}$ results shown in Figure 7.5 that the difference in $S_{13}$ results at 10 GHz for $L$ equal $\lambda/4$ and for $L$ equal $0.42\lambda$ for 2 mm spacing is only 0.4 dB. One may conclude that the length of the coupled line section has a second-order effect on spurious coupling results.

7.3 Coupling Between Two Open-Ends

Computed results for coupling between two open ends in a microstrip line using the multiport network model with distributed sources are shown in Figure 7.6. The line impedance is 50Ω and the substrate has $\epsilon_r = 2.2$ and 10 mil thick. Results in Figure 7.6 present the values of coupling $S_{12}$ when interaction only among the fields at the gap are taken into account. Each edge is represented by one section with a two-dimensional distribution of equivalent magnetic current density. The MCN matrix is $(2 \times 2)$ in size. Coupling values have been computed for three different frequencies and are plotted as a function of the distance between the two edges. One may note that spurious coupling increases for higher frequencies.

Comparison of coupling results ($S_{12}$) for a 15 mm gap in a 50Ω line on
Curve 1: Distributed sources along the edges of the two corners.

Curve 2: Distributed sources along the edges of one corner.

Figure 7.4: Coupling results $S_{13}$ at 40 GHz between two corners of right-angled bends.
Figure 7.5: $S_{13}$ results between two 50Ω right-angled bends on $\varepsilon_r = 2.2$ substrate, 10 mil thick, $L = 5.427\text{mm}$. 

$\varepsilon_r = 2.2$

$h = 0.254\text{nm}$

$z_0 = 50\Omega$

$L = 5.427\text{mm}$
Figure 7.6: Coupling results $S_{12}$ between two open ends.

substrate with $\varepsilon_r = 10.5$, 1.27 mm thick for frequencies ranging from 3.5 GHz to 8.5 GHz are plotted in Figure 7.7. $S_{12}$ results computed using MNM with a two-dimensional distribution of equivalent magnetic current density at the two edges are plotted as curve 1. $S_{12}$ results computed using MNM with a two-dimensional distribution of equivalent magnetic current density at one edge only are plotted as curve 2. $S_{12}$ results computed using MNM with equivalent magnetic current line source at both edges are plotted as curve 3. These results justify the approximation made in deriving equation (5.33).

7.4 Comparison of Coupling Results with Fullwave Analysis Results

In this section computed results for coupling between two open ends, using the MNM, are compared to computed results obtained by using fullwave
Curve 1: Distributed sources along the edges of the two open ends.

Curve 2: Distributed sources along the edge of one open end only.

Curve 3: Equivalent magnetic current line source.

Figure 7.7: Comparison of $S_{12}$ results between two open ends on $\varepsilon_r = 10.5$ substrate, 1.27mm thick.
analysis [18]. Numerical results for coupling, $S_{12}$, between two open ends, on a duroid substrate with $\varepsilon_r = 2.2$ and 10 mil thick, for spacing values from two millimeters ($\sim 8\delta$) to four millimeters ($\sim 16\delta$) are plotted in Figure 7.8. $S_{12}$ results at 20 GHz obtained by using the MNM are shown as curve 1. $S_{12}$ results at 20 GHz using fullwave analysis are shown as curve 2. $S_{12}$ results at 30 GHz using MNM are shown as curve 3. $S_{12}$ results at 30 GHz using fullwave analysis are shown as curve 4. $S_{12}$ results at 40 GHz using MNM and fullwave analysis are plotted as curves 5 and 6 respectively.

Coupling results, $S_{12}$, shown in Figure 7.8 indicate that spurious coupling increases with frequency. One may note that $S_{12}$ results obtained by using fullwave analysis predict a very moderate variation of $S_{12}$ results as function of the spacing between the two open ends. As an example the difference in $S_{12}$ results at 40 GHz, for two millimeters spacing (0.266$\lambda$) and for four millimeters spacing (0.533$\lambda$), is 2.45 dB. However, $S_{12}$ results obtained by using the MNM predicts that $S_{12}$ results decrease faster as a function of the spacing between the two open ends. As an example the difference in $S_{12}$ results at 40 GHz, for two millimeters spacing and for four millimeters spacing, is 8.34 dB.

At 20 GHz and at 30 GHz there is a good agreement between these two sets of results.

7.5 Coupling Between Three Corners of Right-Angled Bends

The numerical results for spurious coupling given in Sections 7.1 and 7.2 were between two discontinuities only. Theoretically the MNM may be applied to investigate spurious coupling effects among a large number of discontinuities. In this section coupling results at 20 GHz between three corners of 50$\Omega$ bends on $\varepsilon_r = 2.2$ substrate, 10 mil thick, are given. The configuration analyzed is shown
Curve 1: MNM 20 GHz.
Curve 2: Fullwave 20 GHz.
Curve 3: MNM 30 GHz.
Curve 4: Fullwave 30 GHz.
Curve 5: MNM 40 GHz.
Curve 6: Fullwave 40 GHz.

Figure 7.8: Comparison of $S_{12}$ results between two open ends with P-Mesh code results.
in Figure 7.9 (inset). $S_{13}$ results are plotted as curve 1, $S_{16}$ results are plotted as curve 2 and $S_{24}$ results are plotted as curve 3. Coupling results shown in this figure present the values of spurious coupling when interactions only among the fields at the outer corners are taken into account.
Figure 7.9: Coupling results between three corners of right-angled bends.
CHAPTER VIII

EXTENSION OF MNM METHOD FOR COUPLED LINE DISCONTINUITIES

8.1 Introduction

Discussions on radiation and spurious coupling produced by discontinuities presented in previous chapters were limited to single microstrip circuits only. Practical microstrip circuits make frequent use of coupled line sections also (for couplers, filter etc.). The basic equivalent magnetic current model for evaluation of external fields as introduced in this thesis is applicable for any circuit including coupled line configurations also. The prerequisite for this modeling approach is a method for evaluating equivalent magnetic current distribution (on the upper surface) representing the fields associated with a coupled line discontinuity, in this case. For a single line, we have used the planar waveguide model which has been previously shown to be valid for the dominant and higher-order modes of a microstrip structure [28]. No similar model had been available earlier for a coupled microstrip line.

In this chapter, we develop a planar-lumped model for characterization of several of the coupled-line junction configurations. The role of this model is similar to that of the planar waveguide model for single microstrip lines used in previous chapters for characterization of several single microstrip discontinuities. The proposed model for coupled lines is a combination of two-
dimensional planar and lumped-element networks. Fields underneath the two strips and those fringing at the outer edges are modeled by planar waveguides. The electric field and magnetic field couplings across the gap are represented by equivalent capacitive and inductive lumped networks. Various model parameters are determined such that $[C]$ and $[L]$ matrices for the model are identical to those for the coupled microstrip section.

Section 8.2 describes the modeling procedure in detail. Model verification for coupled line section is reported in Sections 8.3 and 8.4. Results for chamfered right-angled bend junctions between a coupled microstrip line section and two single microstrip lines are given in Section 8.5 and are compared with experimental results.

Results for a directional coupler with four right-angled bends and with four chamfered bends are given in Section 8.6. In Section 8.7 the effect of discontinuities in a three-section coupler is described. In Chapter 9, the planar-lumped model for a coupled microstrip line is used to evaluate radiation loss and spurious coupling associated with coupled line discontinuities.

8.2 A Planar-Lumped Model for Coupled Microstrip Lines

A coupled microstrip section and the proposed planar-lumped model is shown schematically in Figure 8.1. Fields underneath the two strips and fringing fields at the outer edges $\varepsilon_1$ and $\varepsilon_2$ are represented by two planar waveguides (with magnetic walls) characterized by multiport $Z$-matrices $Z_A$ and $Z_B$ respectively. Fields in the gap between the two lines (including the fringing fields associated with the inner edges) are modeled by the lumped network shown in the figure. Selection of various parameters ensures that the capacitance and inductance matrices for the coupled line configuration and its model are identical.
Figure 8.1: A coupled line section and its planar-lumped model.
8.2.1 Planar Waveguides Parameters

For a single microstrip line, effective width of the equivalent planar waveguide is given by [13]

\[ W_{em}(f) = \eta_0 h / \{ Z_0(f) \sqrt{\varepsilon_{re}(f)} \} \]  \hspace{1cm} (8.1)

where \( \eta_0 \) is the intrinsic wave impedance \((120 \pi \ \Omega)\) of free space and \( h \) is the height of the substrate. \( Z_0(f) \) and \( \varepsilon_{re}(f) \) are the frequency dependent characteristic impedance and effective dielectric constant values obtained from microstrip line analysis. The planar waveguide of height \( h \) and width \( W_{em}(f) \) is bounded by two magnetic walls on the sides and there are no fields outside this region. The medium inside the planar waveguide has a dielectric constant equal to \( \varepsilon_{re}(f) \). This model ensures that \( L \) and \( C \) per unit length of the microstrip line and the model are equivalent. Also, the model has been shown to represent the higher-order microstrip modes accurately. This modeling approach has been successful in analysis of microstrip discontinuities and junctions [28].

For the coupled microstrip line model, the planar waveguides (shown in Figure 8.1) account for fringing fields only at the outer edges. So their effective widths \( W_e(f) \) are chosen to account for outward extension on outer edges only. We get

\[ W_e(f) = \frac{W_{em}(f) + W}{2} \]

where \( W \) is the physical width for the two coupled lines. The dielectric constant for planar waveguide segments is taken to be equal to \( \varepsilon_{re} \) for a microstrip line of width \( W \). The two planar waveguide segments are connected together through a multiport lumped network as shown in Figure 8.1. If there are \( n \) ports on each of the inner edges, each planar waveguide segment is characterized by a \((n + 2 \text{ by } n + 2)\) impedance matrix (assuming that we are considering only one
port at each end of the coupled line section). These $Z$-matrices are derived from
two-dimensional Green's function for rectangular geometry and account for all
the higher ($z$-invariant) two-dimensional modes present in these regions.

8.2.2 C-Network for Modeling E-Field Coupling

The lumped network representing the electric field coupling between the
two strips is derived from the capacitance matrix for the microstrip line. The
capacitance matrix for coupled microstrip lines is obtained from electromagnetic
analysis of coupled microstrips. Analysis algorithm by Bryant and Weiss [41]
was used for computations reported in Section 8.3, 8.5 and 8.6. However, in
order to include the dispersion effects in the planar-lumped model for coupled
line, the spectral domain approach [23] is used to calculate the even and odd
mode characteristic impedance and effective dielectric constants. The spectral
domain approach is reviewed in Appendixes C and D. Even and odd mode
capacitances ($C_e$ and $C_o$) are related to the even and odd mode characteristic
impedances ($Z_{0e}$ and $Z_{0o}$) as follows [21]:

$$C_{e,0} = \frac{1}{\vartheta_{Pe,0} Z_{0e,0}} \quad (8.3)$$

where $\vartheta_{pe,0}$ are the phase velocities for even and odd modes respectively and
are given by

$$\vartheta_{Pe,0} = \frac{c}{\sqrt{\varepsilon_{ree,0}(f)}} \quad (8.4)$$

where $c$ is the velocity of electromagnetic waves in free space. $\varepsilon_{ree,0}(f)$ are the
even and odd mode effective dielectric constants respectively obtained from
spectral domain analysis.

For a purely capacitive multiport network, the $C$-matrix may be defined
as $i = j\omega[C]V$ where $i$ and $V$ are vectors representing port currents and port
voltages respectively. Even and odd mode capacitances ($C_e$ and $C_o$ respectively) are related to the $[C]$ matrix as follows:

$$[C] = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} = \begin{bmatrix} \frac{C_s + C_e}{2} & \frac{C_s - C_e}{2} \\ \frac{C_s - C_e}{2} & \frac{C_s + C_e}{2} \end{bmatrix}$$  \hspace{1cm} (8.5)

A π-network representation of the $[C]$ matrix for a coupled line section of length $\Delta \ell$ is shown in Figure 8.2(a). In the model proposed (Figure 8.1), the shunt capacitance $C_{sh} = C_{11} + C_{12}$ is accounted for partially by the planar segment (with $Z = [Z_a]$). The remaining part of $C_{sh}$ is modeled by the element $C_f$ of the lumped network representing the coupling gap. A section of this network is shown in Figure 8.2(b). $C_f$ is found to be

$$C_f = (C_{sh} - \frac{\varepsilon_0 \varepsilon_r W_e}{h}) \Delta \ell = (C_e - \frac{\varepsilon_0 \varepsilon_r W_e}{h}) \Delta \ell$$  \hspace{1cm} (8.6)

where $\Delta \ell$ is the length of the line represented by the port $i$. The matrix $[C]$ represents capacitances per unit length of the line. The network section shown in Figure 8.2(b) may be represented by the following $Y$-matrix:

$$Y_C = j \omega \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} = j \omega \begin{bmatrix} C_f + C_g & -C_g \\ -C_g & C_f + C_g \end{bmatrix} = j \omega [C_G]$$  \hspace{1cm} (8.7)

When the lumped network of Figure 8.1 has $n$ ports on each side, the complete $C$-matrix $C_G$ may be written as shown in Table 8.1. Non-diagonal terms in each of the four sub-matrices are all zeros.

### 8.2.3 L-Network for Modeling H-Field Coupling

The magnetic field coupling between the two strips is modeled by mutual inductance elements in the lumped network. If we consider two adjacent ports of planar waveguide network A (say 1 and 2) facing the two corresponding ports $(n+1)$ and $(n+2)$ of the segment subnetwork B, the inductive coupling network for this portion may be drawn as shown in Figure 8.3. Values of $L_p$ and $M$ are
Figure 8.2: $C$-network for modeling E-field Coupling.
(a) $\pi$-matrix representation of $[C]$ for coupled lines.
(b) A section of the capacitive part of the lumped network.
calculated by comparing the \([L]\)-matrix of the coupling line with that for the modeling network. The inductance matrix for the coupled line pair is obtained from the capacitance matrix \([C_0]\) for the case when the dielectric is replaced by air. We have the following relation [44] between \([L]\)- and \([C_0]\) matrices:

\[
[L] = \mu_0 \varepsilon_0 [C_0]^{-1} = \begin{bmatrix}
L_{11} & L_{12} \\
L_{21} & L_{22}
\end{bmatrix} \begin{bmatrix}
\text{Henry} \\
\frac{1}{m}
\end{bmatrix}
\]

\hspace{1cm} (8.8)

where \(\mu_0\) and \(\varepsilon_0\) are permeability and permittivity of the free space. The network representation of the inductance matrix in equation (8.8) is shown in Figure 8.3(b). Here \(\Delta \ell\) is the length of the section represented by the partial network shown and \([L]\) is the matrix of inductances per unit length of the coupled line pair. A part of the \(\Delta \ell L_{11}\) is contributed by the inductance of the parallel plate waveguide and the remaining by \(L_p\) included in the lumped network as shown in
Figure 8.3: *L*-network for modeling H-field coupling.
(a) A section of the inductive network in the lumped network.
(b) Network representation for inductance matrix.
(c) Total inductance network.
Figure 8.3(a) and Figure 8.3(c). If the inductance of the parallel plate waveguide region per unit length (= μ₀h/Wₖ) is denoted by \( L_{pt} \), the total inductance network may be drawn as shown in Figure 8.3(c). Equating the admittance matrices of the two networks shown in Figure 8.3(b) and 8.3(c), \( L_p \) and \( M \) are expressed in terms of \( L_{pt} \) and \([L]\) as follows:

\[
L_p = \frac{L_{11} - (L_{11}^2 - L_{12}^2)/L_{pt}}{1 - 2L_{11}/L_{pt} + (L_{11}^2 - L_{12}^2)/L_{pt}^2} \Delta \ell [\text{Henry}] \quad (8.9)
\]

and

\[
M = \frac{L_{12}\Delta \ell}{1 - 2L_{11}/L_{pt} + (L_{11}^2 - L_{12}^2)/L_{pt}^2} [\text{Henry}] \quad (8.10)
\]

In (8.9) and (8.10) it is assumed that \( L_{11} = L_{22} \) and that \( L_{12} = L_{21} \) (symmetric lines). Formulation could, of course, be extended to asymmetric lines also. The inductive part of the complete lumped network portion of the model is shown in Figure 8.4. The \((2n \times 2n)\) admittance matrix of this network is obtained by using Kirchhoff's laws and may be written as follows:

\[
[Y_L] = \frac{-j}{\omega(L_p^2 - M^2)}[L_G] \quad (8.11)
\]

where \([L_G]\) is the \((2n \times 2n)\) inductance matrix shown in Table 8.2. It may be noted that all the four \((n \times n)\) sub-matrices are tridiagonal, all other elements being zero.

In the inductive network shown in Figure 8.4, half inductance sections at the terminal ports \((1, 2, n+1, 2n)\) are not included. A more accurate modeling should incorporate these sections as shown in Figure 8.5. Thus, the lumped network representing the gap becomes a \((2n + 4)\) port network and the \([L_G]\) matrix needs to be modified. The modified matrix is shown in Table 8.3. This modification allows the width \(\Delta \ell\) of each port to be larger and helps in reducing the overall size of the gap matrix.
Figure 8.4: Inductive part of the complete lumped network.

The total lumped network required for modeling the gap is obtained by superposition of capacitance and inductive components. The total admittance matrix of this network is obtained by adding \([Y_C]\) and \([Y_L]\) given by equations (8.7) and (8.11) respectively. We have the admittance matrix of the gap network \([Y_G]\) given by:

\[
[Y_G] = [Y_C] + [Y_L] = j\omega[C_G] - \frac{j}{\omega(L_p^2 - M^2)}[L_G] \tag{8.12.a}
\]

and the corresponding impedance matrix \([Z_G]\) may be obtained as

\[
[Z_G] = [Y_G]^{-1} \tag{8.12.b}
\]

Detailed derivations of \([Y_C]\)- and \([Y_L]\)-matrices are given in Appendix E.
Table 8.2: \(L\)-matrix for the lumped network modeling the gap.

\[
\begin{array}{cccccccccc}
1 & 2 & 3 & \ldots & n-1 & n & n+1 & n+2 & n+3 & \ldots & 2n-1 & 2n \\
L_p & L_p & 0 & \ldots & 2L_p & -L_p & 0 & -L_p & 2L_p & -L_p & \ldots & 2L_p & -L_p \\
-2M & 2M & M & \ldots & -2M & 2M & M & \ldots & -2M & 2M & \ldots & -2M & 2M \\
0 & M & -2M & \ldots & 0 & M & -2M & \ldots & 0 & M & \ldots & 0 & M \\
\end{array}
\]

8.3 Model Verification for Dominant Mode

The planar-lumped model for coupled microstrip lines proposed in Section 8.2 has been verified by considering a quarter-wave section of coupled microstrips, finding its 4-port parameters based on the proposed model, and comparing these with the four-port parameters obtained by conventional coupled line analysis [21]. The starting point for both of these computations is the values of the even and odd mode impedance and effective dielectric constants which are obtained from the Bryant and Weiss algorithm [41].

In conventional coupled-line computations, non-equality of even and odd mode phase velocities has been taken into account for obtaining 4-port \(S\)-parameters. The configuration considered is a pair of microstrip lines (each 50Ω characteristic impedance in isolation) on 0.01 inch thick substrate with \(\varepsilon_r = 2.2\) and their lengths are equal to \(\lambda/4\) (for isolated lines) at 10 GHz. Spacing between
the parallel lines was varied from 0.1 to 20 times the substrate thickness and the coupling parameter $S_{21}$ calculated for various cases.

For computations based on the proposed model, some further details need to be mentioned. For evaluating $Z$-matrices for planar waveguide rectangular segments, single summation formulation [34] has been used. The number of terms needed for convergence was decided by iterative computations to be around 100. Optimum width of the ports interconnecting these rectangular segments to the gap network is found to be $\lambda/88$ (when half inductance sections at terminating ports are not included). Twenty-two ports were taken for $\lambda/4$
length. This number can be reduced to about \( \lambda/50 \) when half inductance sections at 4 ends are included and the \( L \)-matrix formulation shown in Table 8.3 is used. Comparison for the case of coupling coefficient \( S_{12} \) is shown in Figure 8.6.

The two curves coincide and cannot be distinguished in the figure. This very good agreement provides a verification for the proposed model for the dominant even and odd modes.

8.4 Model Verification for Higher-Order Modes

It has been shown [28] that the planar waveguide model for microstrip lines predicts the higher-order modes of microstrip lines fairly accurately. In this section it is shown that the planar-lumped model for coupled microstrips predicts accurately the higher-order modes for coupled microstrip lines. Cut-off frequencies for higher-order modes are evaluated by transverse resonance method.

The planar waveguide model consists of two parallel conductors terminated by magnetic walls (at the outer edges) in the transverse directions. In terms of network theory, a magnetic wall represents an open circuit. In the transverse direction the planar waveguide may be represented as an open-circuited transmission line with length \( W_e(f) \) and effective dielectric constant \( \varepsilon_{re} \) as shown in Figure 8.7. Here \( W_e(f) \) is given by equation (8.2) and

\[
Z_{0T}(f) = \frac{n_0 h}{\Delta \ell \sqrt{\varepsilon_{re}(f)}}
\]  

(8.13)

Using the impedance transformation equation 8.14, the open circuit at the outer edge \( e_1 \) is transformed to the admittance given in equation (8.15) at the inner edge \( e_2 \).

\[
Z_{in} = Z_0 \frac{Z_L + j Z_0 \tan \beta \ell}{Z_0 + j Z_L \tan \beta \ell}
\]  

(8.14)
Table 8.3: \( L \)-network for modeling H-field coupling.

<table>
<thead>
<tr>
<th>( n )</th>
<th>( n+1 )</th>
<th>( n+2 )</th>
<th>( n+3 )</th>
<th>( n+4 )</th>
<th>( n+5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( 2L_p -2L_p ) 0</td>
<td>2M</td>
<td>2M</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>( -2L_p 2L_p ) ( -L_p )</td>
<td>( 2L_p )</td>
<td>3M</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>( -L_p 2L_p ) ( -L_p )</td>
<td>( 0 )</td>
<td>2M 2M</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>( L_p 2L_p ) ( -L_p )</td>
<td>( -L_p )</td>
<td>2M 2M</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( n )</td>
<td>( L_p 2L_p ) ( -L_p )</td>
<td>( L_p 3L_p ) ( -L_p )</td>
<td>( 0 ) ( -2L_p ) ( 2L_p )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( n+1 )</td>
<td>( -L_p ) ( 2L_p ) ( -L_p )</td>
<td>( L_p ) ( 3L_p ) ( -L_p )</td>
<td>( 0 ) ( 2L_p ) ( 2L_p )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( n+2 )</td>
<td>( -L_p ) ( 2L_p ) ( -L_p )</td>
<td>( L_p ) ( 3L_p ) ( -L_p )</td>
<td>( 0 ) ( 2M ) ( -2M )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( n+3 )</td>
<td>( -2M ) ( 2M )</td>
<td>( 2L_p ) ( -2L_p )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( n+4 )</td>
<td>( 2M ) ( 3M ) ( M )</td>
<td>( 2L_p ) ( 3L_p ) ( -L_p )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( n+5 )</td>
<td>( M ) ( -2M ) ( M )</td>
<td>( 0 ) ( -L_p ) ( 2L_p ) ( -L_p ) ( 2L_p )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( 2n )</td>
<td>( -2M ) ( M )</td>
<td>( -L_p ) ( 2L_p ) ( -L_p )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( 2n+1 )</td>
<td>( M ) ( 3M ) ( 2M )</td>
<td>( 2L_p ) ( 3L_p ) ( -2L_p )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( 2n+2 )</td>
<td>( 0 ) ( 2M ) ( -2M )</td>
<td>( 0 ) ( -2L_p ) ( 2L_p )</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Figure 8.6: Comparison of coupling computed by planar/lumped model with conventional coupled line analysis.
where $Z_L$ is the load impedance and $Z_0$ is the characteristic impedance of the transmission line. $\beta$ is the propagation constant and $\ell$ is the line length.

\[ Y_L(e_2) = j \ Y_{0\ell}(f) \tan \beta W_e(f) \tag{8.15} \]

where

\[ Y_{0\ell}(f) = \frac{1}{Z_{0\ell}(f)} \]

and

\[ \beta = k_0 \sqrt{\varepsilon_e} \]

where $k_0$ is the wave number in free space. As described in Section 8.2, fields underneath the two strips and these fringing at the outer edges are modeled by equivalent planar waveguides. Electric and magnetic field couplings in the gap region are modeled by a lumped network. The lumped network representing the electric field coupling between the two strips has been described in Section 8.2.2. The magnetic field coupling between the two strips is modeled by mutual inductance elements as described in Section 8.2.3. The planar-lumped model to predict higher-order modes in coupled microstrip lines is shown in Figure 8.8.

![Figure 8.7: Planar waveguide model of a single microstrip line and its equivalence in the transverse direction.](image-url)
Figure 8.8: Planar-lumped model to evaluate transverse resonance.
The mutual inductance elements may be represented by a $Y$-network as shown in Figure 8.9.

Using Kirchhoff's laws the relations between the voltages and currents in the circuit shown in Figure 8.9(a) are as follows:

$$
\begin{pmatrix}
V_1 \\
V_2
\end{pmatrix} = \begin{pmatrix}
j\omega L_p & j\omega M \\
j\omega M & j\omega L_p
\end{pmatrix} \begin{pmatrix}
I_1 \\
I_2
\end{pmatrix} = [Z_M][I]
$$

(8.16)

Taking the inverse of matrix $[Z_M]$ the following equation is obtained:

$$
\begin{pmatrix}
I_1 \\
I_2
\end{pmatrix} = \frac{1}{j\omega(L_p^2 - M^2)} \begin{pmatrix}
L_p & -M \\
-M & L_p
\end{pmatrix} \begin{pmatrix}
V_1 \\
V_2
\end{pmatrix}
$$

(8.17)

Applying Kirchhoff's laws on the circuit shown in Figure 8.9(b) we obtain

$$I_1 = (Y_{sh} + Y_{se})V_1 - Y_{se}V_2$$

(8.18.a)

$$I_2 = -Y_{se}V_1 + (Y_{sh} + Y_{se})V_2$$

(8.18.b)

Equating terms in equations (8.17) and (8.18) we obtain equations for $Y_{sh}$ and $Y_{se}$ in terms of $M$ and $L_p$,

$$Y_{sh} = \frac{1}{j\omega(L_p + M)}$$

(8.19)

$$Y_{se} = \frac{M}{j\omega(L_p^2 - M^2)}$$

(8.20)

The planar-lumped model to evaluated higher-order modes in coupled lines may be represented as shown in Figure 8.10.

Using even and odd mode analysis, the network representation in the transverse direction for the even and odd modes is shown in Figure 8.11. Part of the fringing fields network representing elements common to even and odd modes may be represented by an additional parallel plate line in the $Z$-direction with $Z_{ef}$, $W_{ef}$ and $\epsilon_{rf}$. However, in the transverse network this line will have a characteristic impedance $Z_{oT}$ given by equation 8.13 (with $\epsilon_{re}$ replaced by $\epsilon_{rf}$), where $Z_{oT}$, $W_{ef}$ and $\epsilon_{rf}$ are given by
Figure 8.9: \( Y \)-network representation of mutual inductance element.
(a) Mutual inductance elements.
(b) \( Y \)-network representation. vskip 2.in
Figure 8.10: Equivalent representation for planar-lumped model to evaluate higher-order modes.

\[
Z_{0f} = \sqrt{\frac{L_p + M}{C_f}}
\]  
(8.21) 

\[
W_{sf} = \frac{\eta_0 h}{Z_{0f} \sqrt{\varepsilon_{rf}}}
\]  
(8.22) 

\[
\varepsilon_{rf} = \frac{c^2(L_p + M)\varepsilon_f}{\Delta \ell^2}
\]  
(8.23)

where

\[
c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}
\]

The transverse resonance for the even and odd mode occurred when the admittance to the left of plane \( e_2 \) is equal to the complex conjugate of the admittance
Figure 8.11: Network representation in the transverse direction for (a) even and (b) odd modes.
to the right of plane \( e_2 \). For even mode this is given by the following transcendental equation:

\[
Y_{0T} \tan\left(\frac{2\pi}{c} f \sqrt{\varepsilon_{r_s}} W_s(f)\right) + Y_{0f} \tan\left(\frac{2\pi}{c} f \sqrt{\varepsilon_{r_f}} W_s(f)\right) = 0
\]  

(8.24)

For odd mode, equation (8.14) is used to find the admittance to the right of point \( e_2 \) where \( Z_0 \) is given by \( Z_{0f} \) and \( Z_L \) is given by

\[
Z_L = \left(\frac{2M}{j\omega (L_p^2 - M^2) + 2j \omega C_s}\right)^{-1}
\]  

(8.25)

The even and odd higher-order mode cut-off frequencies are obtained by using a root searching routine to solve for \( f \) from the transcendental equations derived above.

In order to verify that the model developed in this section predicts higher-order modes in coupled lines, we compared results for the cut-off frequency of higher-order modes obtained using the planar-lumped model approach with results obtained using the spectral domain approach for the analysis for higher-order modes of the coupled lines as described in Appendices C and D. Results for cut-off frequencies for higher-order modes of coupled lines on substrate with \( \varepsilon_r = 9.7 \), 0.635 mm thick with line width of 9.15mm and the spacing between the strips is 0.635 mm are listed in Table 8.4.

It may be noted that there is a fairly good agreement between spectral domain results and the planar-lumped model results. Thus we conclude that the model developed is valid not only for the dominant even and odd modes but also for higher-order even and odd modes. This forms the basis for using the proposed model for characterization of coupled microstrip discontinuities where higher-order modes are also present.
Table 8.4: Cut-off frequency in (GHz) for higher-order modes in coupled lines.

<table>
<thead>
<tr>
<th></th>
<th>Even Mode</th>
<th>Mode EH1</th>
<th>Mode EH2</th>
<th>Mode EH3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Planar</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lumped Model</td>
<td>Odd Mode</td>
<td>5.290</td>
<td>10.588</td>
<td>15.950</td>
</tr>
<tr>
<td>Spectral</td>
<td>Even Mode</td>
<td>5.300</td>
<td>10.700</td>
<td>15.940</td>
</tr>
<tr>
<td>Domain Analysis</td>
<td>Odd Mode</td>
<td>5.100</td>
<td>10.760</td>
<td>15.900</td>
</tr>
</tbody>
</table>

8.5 Comparison with Experimental Results for a Coupled Line Section with Chamfered Bends

The coupled line model proposed above has been applied for characterization of the junctions between coupled line sections and single microstrips located at right angles to the ends of the coupled line section as shown in Figure 8.12(a). The planar-lumped model for this configuration is shown in Figure 8.12(b). In addition to the three segments (two rectangular plus a lumped network) representing the coupled line section, we have three additional planar segments at each of the two upper ports (1 and 2). The analysis procedure for this configuration may be summarized as follows:

1. The number of connections among the various segments is decided based on previous experience/iterative computations. Thirty ports on each side connect the lumped network to planar segments. The number of
Figure 8.12(a): A coupled line with chamfered bends.

interconnections between segments 1 and 2, 2 and 3, and 3 and 4 (and similarly on the right-hand side) is about seven each.

2. Z-matrices for various segments are evaluated from corresponding Green's functions. For isosceles triangles a single summation formulation [35] is used. The Y-matrix for lumped network has been discussed in Section 2.

3. These Z-matrices are combined using the segmentation formula [Appendix B] to obtain a $4 \times 4$ Z-matrix.

4. The $4 \times 4$ Z-matrix is reduced to a $2 \times 2$ matrix with ports 3 and 4 terminated in 50$\Omega$ resistances.

5. The $2 \times 2$ Z-matrix is converted to the corresponding S-matrix.

The results obtained by the above procedure are compared with the
Figure 8.12(b): Planar-lumped model for coupled line configuration of Figure 8.7.
experimental results for the structure fabricated on 4 mil thick GaAs substrate (\(\varepsilon_r = 12.9\)). Comparison for the transmission coefficient \(S_{21}\) is shown in Figure 8.13(a). The curve marked 1 shows computed results, curve 3 shows experimental results, and curve 2 shows the results without taking junction reactance into account. The data for curve 2 is obtained from transmission line analysis of coupled lines.

Results of \(S_{21}\) for the structure shown in Figure 8.11 with spacing of 20\(\mu m\) between the strips are plotted in Figure 8.13(b). In this figure the curve marked 1 shows computed \(S_{21}\) results, curve 3 shows measured \(S_{21}\) results and curve 2 shows computed \(S_{21}\) results without taking junction reactances into account. The differences between measured \(S_{21}\) results and computed \(S_{21}\) results, using the planar-lumped model, at 5.2 GHz and 10 GHz are 0.36 dB and 0.32 dB respectively. The differences between measured \(S_{21}\) results and computed \(S_{21}\) results, using transmission line analysis, at 5.2 GHz and 10 GHz are 1.05 dB and 1.27 dB respectively.

Good agreement between computed and experimental results verifies the modeling procedure developed and its applications to the characterization of coupling line discontinuities.

8.6 Discontinuity Effects in a Single Section Coupler

In order to study the effect of discontinuity on \(S\)-parameters of single section couplers with a nominal couplings of 10 dB and 25 dB, we have used the planar-lumped model to analyze directional couplers with four sharp right-angled bends and also with four chamfered bends. These results and their comparison with an ideal coupler performance (without discontinuities) are discussed in this section.
Figure 8.13(a): Comparison of $S_{12}$ values for coupled line configuration of Figure 8.12 as obtained by: (1) planar model, (2) conventional coupled line analysis, and (3) measurements.
Figure 8.13(b): Comparison of $S_{12}$ values for coupled line configuration of Figure 8.12 as obtained by: (1) planar model, (2) conventional coupled line analysis, and (3) measurements, all for spacing of 20 $\mu$m.
8.6.1 A Directional Coupler with Four Right-Angled Bends

Comparisons of $S_{12}$ and $S_{13}$ (in dB's) for a directional coupler with four right-angled bends with the corresponding values in an ideal coupler are shown in Figures 8.14 and 8.15. One may note that for spacing ($s/h$) values from 0.5 to 18.69 differences in coupling values ($S_{12}$) in the two cases are around 0.1 dB to 0.3 dB.

The values of the isolation $S_{13}$ (for the same spacing) differ as much as 2 dB to 4 dB. It is found that the isolation $S_{13}$ for the coupled-line with four right-angled bends is lower (better). These differences may be viewed as being caused by changes in effective lengths of the coupled line section due to discontinuity effects. This point is explained later in Section 8.6.3.

For all spacing ($s/h$) values from 0.5 to 19.69, the reflection coefficient $S_{11}$ for the straight coupled line (ideal) is better than that for the coupler with four right-angled bends, by amount as much as 6 dB to 25 dB as shown in Figure 8.16.

8.6.2 A Coupler with Four Chamfered Bends

A comparison of coupling $S_{12}$ (in dB) for an ideal coupler and for two cases of couplers with chamfered bends with different length of the coupled section is shown in Figure 8.17. In this figure, curve 1 is for the ideal coupler, curve 2 is for a coupler with chamfered bends when the distance between the ends of the two bends is a quarter wavelength, and curve 3 is for the coupler with chamfered bends when the distance between the center lines of the two single microstrips is a quarter wavelength, as shown in Figure 8.17. For spacing values ranging from $s/h = 0.5$ to $s/h = 8$, the differences between curves 1 and 2 are around 0.1 dB to 0.3 dB. The differences between curves 1 and 3 are
Figure 8.14: Comparison of $S_{12}$ values for coupled line with four 90° bends and an ideal coupled lines section.
Figure 8.15: Comparison of $S_{13}$ values for an ideal coupled line section and a coupled line section with four right-angled bends.
Figure 8.16: Comparison of $S_{11}$ values in dB for an ideal coupled line section and a coupled line with four 90° bends.
around 0.1 dB to 1.8 dB. These differences may be attributed to changes in the effective lengths of the coupled line sections. In order to demonstrate that the effect of the discontinuity reactances leading to shift in the center frequency and to a change in the effective length of the coupled line section, the S-parameters for the chamfered bends coupler were calculated for frequencies ranging from 8 GHz to 12 GHz. These are shown in Figures 8.18 to 8.25 and are discussed in the next subsection.

8.6.3 Discontinuity Effects in a 10 dB and 25 dB Couplers

In order to study the effect of discontinuities on the S-parameters of a single section λ/4 coupler, two cases are considered: a 10 dB coupler (s/h) = 0.05 and a 25 dB coupler (s/h = 2). It is of interest to know the effects of the discontinuity on S-parameters for low and high coupling as a function of the length of the coupled line section and as a function of frequency. We calculated the S-parameters for frequencies ranging from 8 GHz to 12 GHz for the ideal coupler, for a coupler with four right-angled bends and a coupler with four chamfered bends. Results are given, for 10 dB and 25 dB coupler, for two cases when the distance between the ends of the two bends is λ/4 and when the distance between the center lines of the two single microstrips is λ/4 as shown in Figure 8.18 (inset).

The S-parameters were calculated at the reference planes of the discontinuity (r, r as shown in Figure 8.18). When the distance between the center line of the two single microstrips is λ/4, the difference in coupling $S_{12}$ between a coupler with four right-angled bends and an ideal coupler are around 0.3 dB for the 10 dB coupler and around 0.6 dB for the 25 dB coupler for frequencies ranging from 8 GHz to 12 GHz, as shown in Figures 8.18 and 8.19. For the
Figure 8.17: Comparison of coupling values for the three configurations shown.
Figure 8.18: Comparison of $S_{12}$ in dB for a 10 dB coupler for the three configurations shown.
Figure 8.19: Comparison of $S_{12}$ in dB for a 25 dB coupler for the three configurations shown.
Figure S.20: Comparison of $S_{12}$ in dB for the three configurations shown, 10 dB couplers.
Figure 8.21: Comparison of $S_{12}$ in dB for the three configurations shown, 25 dB couplers.
Figure 8.22: Comparison of $S_{14}$ phase in degrees ($\lambda/4$ between center line of bends), for 10 dB couplers.
Figure 8.23: Comparison of $S_{14}$ phase in degrees ($\lambda/4$ between center line of bends) for 25 dB couplers.
Figure 8.24: Comparison of $S_{14}$ ($\lambda/4$ between ends of bends) for the three configurations shown (phase evaluated with respect to the four reference planes shown in Figure 8.23), for 10 dB couplers.
Figure 8.25: Comparison of $S_{14}$ ($\lambda/4$ between ends of bends) for the three configurations shown (phase evaluated with respect to the four reference planes shown), for 25 dB couplers.
same case, the differences in $S_{12}$ between an ideal coupler and a coupler with four chamfered bends are 1.25 dB for the 10 dB coupler and 1.6 dB for the 25 dB coupler at 8 GHz. At 12 GHz, the differences are 0.13 dB for the 10 dB coupler and 0.1 dB for the 25 dB coupler. Figures 8.22 through 8.25 show the phase of transmission $S_{14}$ in degrees for these cases, and we note that at 12 GHz the phase of $S_{14}$ for the coupler with four chamfered bends is 120.04° for the 10 dB coupler and 90.95° for the 25 dB coupler.

A similar comparison is given for the case when the distance between the ends of the two bends is $\lambda/4$. In this case, the differences in $S_{12}$ between an ideal coupler and a coupler with four sharp 90° bends are 0.6 dB for the 10 dB coupler and 1.6 dB for the 25 dB coupler at 8 GHz, and 1.25 dB for the 10 dB coupler and 0.8 dB for the 25 dB coupler at 12 GHz. For this case the differences in $S_{12}$ values between an ideal coupler and a coupler with four chamfered bends are 0.26 dB for the 10 dB coupler and 0.5 dB for the 25 dB coupler at 8 GHz. At 12 GHz, for the same case, the differences are 0.4 dB for the 10 dB coupler and 0.3 dB for the 25 dB coupler, as shown in Figures 8.20 and 8.21. One may conclude that when the coupling is low, the discontinuity effects are more significant, because the coupling values and the discontinuity effects are of the same order.

8.6.4 Comparison with Fullwave Analysis for a Coupler with Four Chamfered Bends

In this section, the planar-lumped model results are compared to full-wave analysis results, P-Mesh code [18]. The planar-lumped model proposed in this section and the P-Mesh code are used to calculate the $S$-parameters of a one-section coupler with four chamfered bends on a substrate with $\varepsilon_r = 2.2$, 10 mil thick. The length of the coupled section is $\lambda/4$ at 10 GHz and the spacing
between the parallel strips is 0.5 mil. $S_{12}$ results are plotted in Figure 8.26 for frequencies ranging from 8 GHz to 12 GHz.

$S_{12}$ results using the planar-lumped model are plotted as curve 1. $S_{12}$ results using the P-Mesh code are plotted as curve 2. $S_{12}$ results using the P-Mesh code are obtained by taking three cells along the line width and five cells along the coupled line section. The total number of cells is 252. The cpu time needed to analyze this circuit at one frequency is 50 minutes. However, the cpu time needed to analyze this circuit using the PLM is around 1 minute. $S_{12}$ results using quasistatic analysis (taking no discontinuity effects into account) are plotted as curve 3. The disagreement in $S_{12}$ results between PLM and the P-Mesh code is around 0.8 dB to 1 dB. $S_{12}$ results shown in Figure 8.26 indicate that discontinuity effects on couplers cannot be neglected.

8.7 Discontinuity Effects in a Three-Section Coupler

A 10 dB, three-section coupler was designed at 10 GHz on 4 mil Gallium Arsenide substrate ($\varepsilon_r = 12.9$). The values of the even and odd mode capacitance ($C_e$) and ($C_o$), and even and odd mode effective dielectric constants $\varepsilon_{re}$ and $\varepsilon_{ro}$ were calculated using the Bryant and Weiss program [41]. The coupler design was carried out using the procedure given in [45]. The configuration of the three-section coupler is shown in Figure 8.27(a). The three-section coupler was broken down into regular elementary segments as shown in Figure 8.27(b). Segment 1 is a one-section straight coupler; segment 2 is an isosceles triangular segment; segment 3 is a rectangular segment; segment 4 is an isosceles triangular segment; and segment 5 is again a straight coupler. Segments 6 to 9 are the corresponding segments near port 4. The $Z$-matrices for these segments are the same as for segments 1 through 4 and need not be computed again. $Z$-matrices
Figure 8.26: Comparison of $S_{12}$ results with fullwave analysis for a coupler with four chamfered bends.
for each segment are computed by using the procedures described in Sections 8.1 through 8.3.

The results calculated by the planar-lumped model are compared with conventional analysis (no discontinuity effects included) using transmission line analysis and with Touchstone Program [46] taking discontinuity effects into account [47, 48]. The variation of the coupling factor $S_{12}$ with frequency is shown in Figure 8.28. $S_{12}$ results using the planar-lumped model are shown as curve 1 of this figure. $S_{12}$ results obtained using transmission line analysis are shown as curve 2. Touchstone computer program [46] results taking discontinuity effects into account are shown as curve 3. The difference in $S_{12}$ between the planar-lumped model (curve 1), and transmission line analysis (curve 2) at 7 GHz is 0.4 dB and 0.75 dB at 10 GHz. The difference between results shown in curve 1 and curve 3 at 7 GHz is 0.03 dB and 1.1 dB at 13 GHz.

The differences in isolation values $S_{13}$ between the planar-lumped model and transmission line analysis are 2 dB at 7 GHz and 20 dB at 13 GHz. The variation of the reflection coefficient $S_{11}$ with frequency is shown in Figure 8.29. The differences between $S_{11}$ results obtained using the planar-lumped model (curve 1) and transmission line analysis (curve 2) are 1 dB at 7 GHz and 3 dB at 13 GHz. The differences between curve 1 and the Touchstone program results (curve 3) are 2 dB at 7 GHz and 3.2 dB at 13 GHz. Comparison of transmission $S_{14}$ is shown in Figure 8.30. The differences in $S_{14}$ between the proposed model (curve 1) and transmission line analysis (curve 2) are 0.06 dB at 7 GHz and 0.27 dB at 13 GHz. The differences in $S_{14}$ between curve 1 and Touchstone results (curve 3) are 0.12 dB at 7 GHz and 0.3 dB at 13 GHz.
Figure 8.27: A 10 dB three-section coupler.
Figure 8.28: Comparison of $S_{12}$ results for a three-section coupler.
Figure 8.29: Comparison of $S_{11}$ results for a three-section coupler.
Figure 8.30: Comparison of $S_{14}$ results for a three-section coupler.
CHAPTER IX

RADIATION LOSS AND PARASITIC COUPLING CAUSED BY COUPLED MICROSTRIP LINE DISCONTINUITIES

9.1 Introduction

The planar-lumped model for coupled microstrip line discontinuities discussed in Chapter 8 is used to evaluate radiation loss and parasitic coupling between coupled microstrip discontinuities. The procedure is similar to that used in Chapters 3 and 5 for single microstrip line discontinuities.

Since no earlier results for power radiated from coupled microstrip line discontinuities are available in literature, a comparative assessment cannot be carried out.

The planar-lumped model and the multiport network model are used to evaluate radiation loss and parasitic coupling between coupled microstrip line discontinuities. Results of radiation loss from a coupled line section with four chamfered bends are given in Section 9.3.

As in the case of single microstrip discontinuities, radiation from coupled microstrip line discontinuities generates undesired interaction between different parts of the circuit due to external electromagnetic coupling. This radiation is coupled to other parts of the circuit and may result in degradation in circuit performance. In Section 9.4 we describe the approach used to evaluate parasitic coupling caused by coupled microstrip line discontinuities, and some typical
results are given.

9.2 Evaluation of Radiation Loss from Coupled Microstrip Line Discontinuities

In order to evaluate the external fields produced by a coupled microstrip line discontinuity, we first obtain voltages at the edges of the discontinuity configuration. A multiport network model similar to that developed for single microstrip discontinuities, as described in Chapter 2, is employed. For implementing this method we add a number of open ports at the edges of the coupled line discontinuity structure from which radiation loss is being evaluated. The circuit behavior is simulated by terminating ports 2, 3 and 4 in a matched load and adding a matched source to port 1, as shown in Figure 9.1(a). Voltages at the $N$ ports at the edges are computed by using a similar procedure as described in Section 2.3. The only distinction in this case is the use of the planar-lumped model for coupled lines to evaluate voltages at the coupled line edges. Voltages in the gap region may be represented by two equivalent magnetic line sources along each inner edge or by a single equivalent magnetic line source. When two magnetic line sources are used, the amplitude $M$ of the magnetic current is twice that of the edge voltage at that location, and the phase of $M$ is equal to the phase of the corresponding edge voltage. When a single magnetic line source is used, the amplitude and phase of the magnetic line source are obtained by subtracting the complex value of the voltages at opposite ports in the gap region and multiplying the result by two. The total radiation is computed using the superposition of the far field radiation by each section using the equations given in Section 2.6. The differences in computed results of power radiated from coupled microstrip line discontinuities obtained by representing the edge voltages in the gap region by a single magnetic line source or by two magnetic
line sources are negligible.

9.3 Power Radiated From a Coupled Line with Four Chamfered Bends

A coupled line section with four chamfered bends on \( \varepsilon_r = 2.2 \) substrate (0.01 inch thick) with spacing of 0.005 inch between the parallel strips is shown in Figure 9.1. Results for power radiated from this coupler (normalized to the input power) are plotted in Figure 9.2 for frequencies ranging from 8 GHz to 14 GHz. The radiated power at 8 GHz is -29.89 dB and at 12 GHz the radiated power is -18.13 dB. At 14 GHz the radiated power decreases to -20.05 dB. The power radiated from two chamfered bends connected to a 50\( \Omega \) microstrip line is shown as curve 1 in Figure 9.2. At 10 GHz the radiated power from the coupled line section with four chamfered bends is 0.84 dB higher than the power radiated from two chamfered bends connected to a 50\( \Omega \) line, whereas at 12 GHz the coupled line configuration radiates 8.6 dB more. Also, radiation from the coupled line configuration decreases as frequency is increased from 12 GHz to 14 GHz, whereas for single line bends it continues to increase monotonically.

It is of interest to investigate the radiated power from coupled microstrip line discontinuities as a function of the spacing between the strips in the coupled line section. Results for the power radiated at 10 GHz from a coupled line section with four chamfered bends on \( \varepsilon_r = 2.2 \) substrate 10 mil thick, for spacing values \( (s/h) \) from 0.05 to 12 are plotted in Figure 9.3 as curve 2. The power radiated at 10 GHz from two chamfered bends connected to a 50\( \Omega \) microstrip line are plotted as curve 1 in this figure. As expected, we note that when the coupling value is less than -45 dB \( (s/h \sim 12) \), the radiated power from the coupled line section with four chamfered bends is the same as the radiated power from two chamfered bends connected to a 50\( \Omega \) microstrip line. One may note that
Figure 9.1: Multiple ports located at the edges of a coupled microstrip line discontinuity.
(a) Ports around the edges of the coupled line circuit (only one half of the configuration shown).
(b) A coupled microstrip line section with four chamfered bends.
\[ \varepsilon_r = 2.2 \]

\[ h = 0.254\text{mm} \]

\[ S/H = 0.05 \]

**Figure 9.2:** Radiated power from a coupled line with four chamfered bends.
for spacing values \((s/h)\) from 0.05 to 4 the power radiated increases slightly. The reason for this phenomenon is that as the spacing increases, the radiating aperture increases and the coupling values are around -30 dB, about the same level as the power radiated. For these spacing values the contribution to the power radiated is from an array of four chamfered bends.

![Graph](image)

Figure 9.3: Radiated power from a coupled line section with four chamfered bends as a function of spacing.

### 9.4 Parasitic Coupling Caused by Coupled Microstrip Line Discontinuities

A planar multiport network model of the microstrip discontinuity configuration, the planar-lumped model for coupled line sections and the segmentation method are used to evaluate voltage distribution around the edges of the coupled microstrip line discontinuity. As in Section 5.3, this voltage distribution
is expressed as an equivalent magnetic current line source distribution which is used to calculate the induced currents at the location of the other discontinuity for spurious coupling calculations. This is similar to the procedure described in Section 5.2. The only distinction in this case is that the planar-lumped network model for coupled lines is used to evaluate voltages and coupling in the coupled line section. In this chapter we use a combination of dynamic external coupling network (MCN) with a planar-lumped model for coupled sections (which accounts for electrostatic (capacitive) and magnetic coupling), as shown in Figure 9.4. An implementation of this procedure for coupling between a coupled microstrip section with chamfered bends and a single microstrip with a chamfered bend is shown in Figure 9.5.

![Diagram](image)

Figure 9.4: Network model for evaluating coupling between microstrip discontinuities associated with coupled line sections.
9.5 Computational Details for Evaluation of Spurious Coupling between Coupled Microstrip Line Discontinuities

Computational details given earlier in Chapters 3, 5 and 8 for evaluation of the MCN and PLM are also applicable for the evaluation of spurious coupling between coupled microstrip line discontinuities. However, it is of interest to investigate the convergence of numerical results as a function of $X_0$, the width of the magnetic current distribution.

Convergence of $S$-parameters as a function of $X_0$ was investigated at 10
Table 9.1: S-parameters for a coupled line section with two right-angled bends for two values of $X_0$ for 2.5mm spacing between the strips.

<table>
<thead>
<tr>
<th></th>
<th>$X_0 = 1.5\ h$</th>
<th>$X_0 = 2.5\ h$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_{11}(\text{dB})$</td>
<td>-20.18</td>
<td>-20.18</td>
</tr>
<tr>
<td>$S_{12}(\text{dB})$</td>
<td>-1.48 $10^{-2}$</td>
<td>-1.43 $10^{-2}$</td>
</tr>
<tr>
<td>$S_{13}(\text{dB})$</td>
<td>-43.17</td>
<td>-42.97</td>
</tr>
<tr>
<td>$S_{14}(\text{dB})$</td>
<td>-50.29</td>
<td>-51.49</td>
</tr>
</tbody>
</table>

GHz for a coupled line section with two right-angled bends on $\varepsilon_r = 2.2$ substrate, 10 mil thick.

Comparison of S-parameters for spacing of 2.5 mm ($s/h = 10$) and 3 mm ($s/h = 12$) between the parallel strips for different values of $X_0$ are given in Tables 9.1 and 9.2. One may note that there is almost absolute convergence in reflection, transmission and coupling coefficients. One may conclude that choosing $X_0$ to be equal to twice the height of the substrate provides a good estimate of spurious coupling between coupled line discontinuities.

Table 9.2: S-parameters for a coupled line section with two right-angled bends for two values of $X_0$ for 3mm spacing between the strips.

<table>
<thead>
<tr>
<th></th>
<th>$X_0 = 1.5\ h$</th>
<th>$X_0 = 2.5\ h$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_{11}(\text{dB})$</td>
<td>-20.18</td>
<td>-20.18</td>
</tr>
<tr>
<td>$S_{12}(\text{dB})$</td>
<td>-1.41 $10^{-2}$</td>
<td>-1.41 $10^{-2}$</td>
</tr>
<tr>
<td>$S_{13}(\text{dB})$</td>
<td>-46.07</td>
<td>-45.97</td>
</tr>
<tr>
<td>$S_{14}(\text{dB})$</td>
<td>-51.86</td>
<td>-52.43</td>
</tr>
</tbody>
</table>
9.6 Parasitic Coupling between a Coupled Line Section with Two Right-Angled Bends

In order to calculate $S$-parameters at 10 GHz for a quarter wavelength single coupled line section connected to two right-angled bends on $\varepsilon_r = 2.2$ substrate and 10 mil thick, a planar lumped network is connected between the parallel strips and a mutual coupling network is connected between the two right-angled bends as shown in Figure 9.6. Coupling results $S_{13}$ are shown in Figure 9.7 for $s/h$ values greater than $\sim 10$. $S_{13}$ results shown as curve 1 of this figure are for a coupled line section without the right-angled bends. Results for a coupled line section with two right-angled bends are shown in curve 2 of this figure. One may conclude that parasitic coupling due to radiation from right-angled bends cannot be neglected in the analysis of coupled microstrip line discontinuities. As an example, the coupling coefficient $S_{12}$ for the ideal coupler with spacing of 1.7 mm is -37.47 dB and $S_{12}$ for the coupled line section with two right-angled bends is -37.02 dB.

9.7 Spurious Coupling between a Coupler with Four Chamfered Bends and a Right-Angled Bend

The multiport network model for coupling and the planar-lumped model for coupled parallel sections are used to evaluate spurious coupling at 10 GHz between a coupler and a right-angled bend, as shown in Figure 9.8, on $\varepsilon_r = 2.2$ substrate, 10 mil thick. The spacing between the parallel strips in the coupled line section is equal to 3.15 times the height of the substrate. As shown in the inset of Figure 9.8, there are 24 ports along the edges of the right-angled bends. Curve A in this figure presents results for $S_{12}$ when no ports are taken along the edges of the chamfered bend edges of the coupler as shown in the inset A. Curve B is for $S_{12}$ results with no ports along the single 50Ω microstrip line connected
Figure 9.6: Multiport network model for incorporating coupling and electrostatic coupling between two bends connected to a coupled line section.

to the coupled line section as shown in the inset B. \( S_{12} \) results of the coupled line section with four chamfered bends without spurious coupling effects are plotted as curve C. Curve D gives results of spurious coupling with ports along the edges of the 50Ω single microstrip line and ports along the chamfered bend edges of the coupler. One may note that for spacing values (s) between the right-angled bend and the coupler, greater than 12 times the substrate height \( S_{12} \) results in curves A, B and D converge to \( S_{12} \) results of a coupled line with four chamfered bends (curve C). Thus, one may conclude that for normalized spacing values to the substrate height greater than 12, spurious coupling between a right-angled bend and a coupled line section with four chamfered bends is negligible. The differences in \( S_{12} \) between curves A, B, D and C are listed in Table 9.3.
Figure 9.7: $S_{13}$ for a coupled section with two right-angled bends.

Table 9.3: Differences in $S_{12}$ between curves A, B, D and curve C.

<table>
<thead>
<tr>
<th>Spacing (s/h)</th>
<th>Difference in dB between A-C</th>
<th>Difference in dB between B-C</th>
<th>Difference in dB between D-C</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>0.19</td>
<td>-0.969</td>
<td>-4.3</td>
</tr>
<tr>
<td>6</td>
<td>0.085</td>
<td>-0.37</td>
<td>-0.52</td>
</tr>
<tr>
<td>8</td>
<td>0.06</td>
<td>-0.17</td>
<td>-0.16</td>
</tr>
<tr>
<td>12</td>
<td>0.02</td>
<td>-0.06</td>
<td>-0.16</td>
</tr>
</tbody>
</table>

Parasitic coupling ($S_{13}$) results are plotted in Figure 9.9. Curve A of this Figure presents $S_{13}$ results obtained without ports along the chamfered bend edges of the coupler. $S_{13}$ results of curve B are obtained without ports along the single microstrip line connected to the coupled line section. $S_{13}$ results in curve
Figure 9.8: Effect of a right-angled bend in proximity on coupled $S_{21}$ in a coupled line section.
C are obtained with ports along the single microstrip line connected to the coupled line section and ports along the chamfered bend edges of the coupler.

One may note that the differences in $S_{13}$ between curve B and curve C are much smaller than the differences in $S_{13}$ results between curves A and C. These results indicate that the ports along the chamfered bends have greater contribution to the spurious coupling than ports along the single 50Ω microstrip line connected to the coupled line section.

One may conclude that parasitic coupling effects caused by coupled microstrip line discontinuities are not always negligible ($\sim 30$ dB for $s/h = 3.15$).

9.8 Spurious Coupling between a Coupler with Four Chamfered Bends and a Chamfered Bend for Different Orientations of the Chamfered Bend

Here, the procedure developed is used to evaluate spurious coupling at 10 GHz between a coupler with four chamfered bends and a chamfered bend on $\varepsilon_r = 2.2$ substrate 10 mil thick. The configurations are shown in Figures 9.10 and 9.11. $S_{12}$ results for spacing values (normalized to the substrate height) from 4 to 12 are plotted in Figure 9.10. In this figure, $S_{12}$ results are given for various locations of the chamfered bend with respect to the coupled section, as shown in the inset of this figure. Curve A of Figure 9.10 shows $S_{12}$ results of a coupled line section with four chamfered bends without spurious coupling effects. One may note that for spacing values ($s/h$) greater than 12 times the substrate height, $S_{12}$ results with spurious coupling effects converge to $S_{12}$ results for a coupled line with four chamfered bends (curve A).

The differences in $S_{12}$ between curves B, C and A are listed in Table 9.4. $S_{13}$ results for the same cases are plotted in Figure 9.11. $S_{13}$ results for spacing values ($s/h$) values 4 and 12 are -29.54 and -35.5 respectively for curve A.
Figure 9.9: $S_{13}$ results between a coupler and a right-angled bend.
Figure 9.10: $S_{12}$ results between a coupler and a chamfered bend.
Table 9.4: Differences in spurious coupling $S_{12}$ between curves B, C and curve A.

<table>
<thead>
<tr>
<th>Spacing (s/n)</th>
<th>Difference in dB between curves B, A</th>
<th>Difference in dB between curves A, C</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>-0.38</td>
<td>0.27</td>
</tr>
<tr>
<td>6</td>
<td>-0.14</td>
<td>0.2</td>
</tr>
<tr>
<td>8</td>
<td>-0.08</td>
<td>0.16</td>
</tr>
<tr>
<td>12</td>
<td>-0.03</td>
<td>0.08</td>
</tr>
</tbody>
</table>
Figure 9.11: $S_{13}$ results between a coupler and a chamfered bend.
CHAPTER X

THE EFFECTS OF ENCLOSURES ON PARASITIC COUPLING BETWEEN MICROSTRIP DISCONTINUITIES

10.1 Package Effects on Microstrip Performance

Microstrip circuits are often enclosed in a metallic enclosure to protect the circuit from environment and to provide electrical isolation between different parts of the subsystem. Metallic enclosures can support several modes which modify the coupling between various parts of the circuit and consequently alter the circuit performance.

The effects of a metallic enclosure on the transmission parameters of microstrip lines are summarized in [10]. As a conducting cover is moved nearer to a microstrip line the characteristic impedance and the effective permittivity are reduced. Similar effects are observed if conducting side walls are moved nearer to a microstrip line. Usually, if the distance of the microstrip line to the package walls is greater than four times the substrate height the effect on the microstrip line parameters is negligible.

In [49] a numerical algorithm is presented to find the resonant frequency and quality factor of the lowest order resonant mode in a metal enclosure. These resonant cavity modes may be damped by placing a dielectric sheet coated with a resistive film in the cavity. In [50], a moment method formulation is used to analyze microstrip circuits in a lossy enclosure.
In this chapter, the effect of enclosures on parasitic coupling between microstrip discontinuities is analyzed. This analysis is based on modal expansion of electromagnetic fields in cavities [51, 52] and the multiport network model for evaluation of parasitic coupling between microstrip discontinuities as described in Chapter 5. In [51] a similar method involving modal expansion of fields in cavities and the multiport network model has been used to analyze electromagnetically fed microstrip patches.

10.2 Method of Analysis

A planar multiport network model of the discontinuity configuration and the segmentation method are used to evaluate voltage distribution around the edges of the discontinuity configuration (ignoring the enclosure). This voltage distribution is expressed as an equivalent magnetic current line source distribution which is used to calculate the magnetic fields inside the rectangular package, using modal expansion in the enclosure, at the location of the other discontinuity for spurious coupling calculations. An equivalent magnetic current modeling of two right-angled bends in an enclosure is shown in Figure 10.1. The system of coordinates used in this chapter and a magnetic current element source in a rectangular enclosure are shown in Figure 10.2. The spurious coupling between two discontinuities (due to external fields in the package) may be incorporated in the multiport network model by connecting an additional multiport network between the two discontinuities as shown in Figure 10.3. The coupling network MCN is characterized in terms of an admittance matrix \([Y_m]\). Elements of this matrix represent mutual admittances between various sections of the edges of the two discontinuities.

A procedure similar to that described in Section 5.2 is used to calculate
spurious coupling between discontinuities in a rectangular enclosure. The main modification is that the magnetic fields are calculated using modal expansion in cavities as described in [52]. \( E \) and \( H \) fields inside a rectangular enclosure excited by a magnetic current \( M \) may be written as

\[
E = \sum_i \frac{j \omega_i E_i}{\omega_i^2 - \omega_i^2} \int V M \cdot H_i^* \, dv
\]

(10.1)

\[
H = \sum_i \frac{j \omega_i H_i}{\omega_i^2 - \omega_i^2} \int V M \cdot H_i^* \, dv
\]

(10.2)

where \( E_i \) and \( H_i \) are normalized mode vectors that represent the source-free electrical and magnetic fields for the \( i \)-th mode in the cavity and \( \omega \) is the operating frequency (in radians/sec), \( \omega_i \) is the mode frequency given by

\[
\omega_i^2 = \frac{1}{\mu_\varepsilon} (\alpha_1^2 + \alpha_2^2 + \alpha_3^2)
\]

(10.3.a)
Figure 10.2: A magnetic current element source in a rectangular enclosure.

where $\alpha_1$, $\alpha_2$ and $\alpha_3$ are the mode coefficients and are given by

$$\alpha_1 = \frac{m\pi}{a}$$

$$\alpha_2 = \frac{n\pi}{b}$$

$$\alpha_3 = \frac{p\pi}{h}$$

(10.3,b)

The integration is carried over the volume $v$ of the enclosure.

Equations 10.1 and 10.2 may be written as

$$E = \sum_{i=1}^{\infty} B_{te} E_i$$

(10.4)

$$H = \sum_{i=1}^{\infty} B_{ih} H_i$$

(10.5)

where $B_{te}$ and $B_{ih}$ are the source terms given by

$$B_{te} = \frac{j}{\omega^2 - \omega_i^2} \int_v M \cdot H_i^* \, dv$$

(10.6)
Figure 10.3: Circuit with equivalent network model of parasitic coupling in an enclosed environment.

\[ B_{th} = \frac{j \omega}{\omega^2 - \omega_i^2} \int v M \cdot H_i^* dv \]  \hspace{1cm} (10.7)

The boundary conditions at six perfect conducting walls are

\[ \begin{align*}
E_{\text{tan}} &= 0 \\
H_{\text{norm}} &= 0 \\
\end{align*} \hspace{1cm} \text{at the walls} \hspace{1cm} (10.8) \]

When these boundary conditions are applied, source-free electric and magnetic fields \( E_i, H_i \) are obtained as [53]

\[ H_i = a_x \ H_x + a_y \ H_y + a_z \ H_z \]

where

\[ \begin{align*}
H_x &= H_1 \sin(\alpha_1 x) \cos(\alpha_2 y) \cos(\alpha_3 z) \\
H_y &= H_2 \cos(\alpha_1 x) \sin(\alpha_2 y) \cos(\alpha_3 z) \\
H_z &= H_3 \cos(\alpha_1 x) \cos(\alpha_2 y) \sin(\alpha_3 z) \\
\end{align*} \hspace{1cm} (10.9) \]

and

\[ E_i = a_x \ E_x + a_y \ E_y + a_z \ E_z \]
where

\[ E_x = E_1 \cos(\alpha_1 x) \sin(\alpha_2 y) \sin(\alpha_3 z) \]
\[ E_y = E_2 \sin(\alpha_1 x) \cos(\alpha_2 y) \sin(\alpha_3 z) \]
\[ E_z = E_3 \sin(\alpha_1 x) \sin(\alpha_2 y) \cos(\alpha_3 z) \]

(10.10)

The amplitudes \( H_1 \) through \( E_3 \) are given in Appendix G.

The source term and the source-free fields are combined and summed over all the cavity modes to obtain the total electric and magnetic fields at any given location \((X_0, Y_0, Z_0)\). The magnetic field induces a current in the circuit portion at that location. The induced current density is expressed as

\[ J = -n \times H \]

(10.11)

The value of the \( Y_{ij} \) element in the admittance matrix (for the spurious coupling network) is obtained from the current induced on the \( j \)-th subsection of the circuit edge as a result of a voltage \( v_i \) at the \( i \)-th subsection. We have

\[ Y_{ij} = J_j \frac{d\ell_j}{v_i} \]

(10.12)

where \( d\ell_j \) is the width of the \( j \)-th port. The \( Z \)-matrix of the mutual coupling network is the inverse of the matrix \([Y_m]\). Segmentation method is used to combine the \( Z \)-matrix representation of the discontinuities and that of the coupling network to yield the overall \( Z \)-matrix. The resulting \( Z \)-matrix is converted to \( S \)-matrix. The effect of the coupling on the circuit performance is obtained from these \( S \)-parameters. The procedure for evaluating parasitic coupling between two components \( A \) and \( B \) in a rectangular enclosure (Figure 10.3) may be summarized as follows: As described in Subsection 2.3.3, component \( A \) is broken down into elementary regular segments, connected together at the interfaces by a discrete number of interconnections. \( N \) ports are added at the edges of the discontinuity structure from which the spurious coupling is being evaluated.
The $Z$-matrix of component $A$, $Z_A$, is evaluated by using the Green's function approach and the segmentation formula. In a similar manner the $Z$-matrix of component $B$, $Z_B$, is evaluated.

The fringing fields at the edges of component $A$ are represented by small sections of equivalent magnetic current strips of length $dt$.

Fields inside the enclosure produced by the equivalent magnetic current at the $i$-th subsection of component $A$ are calculated by using modal expansion of fields in an enclosure. The induced current in the $j$-th subsection of component $B$ is calculated using equation 10.11. The $Y_{ij}$ element of the mutual coupling admittance matrix is calculated using equation 10.12.

The above procedure is repeated to compute all the elements of the coupling matrix $[Y_m]$. The matrix $[Y_m]$ is inverted to obtain the matrix $[Z_C]$. The segmentation method is used to combine $[Z_A]$ with $[Z_C]$ to produce $[Z_{AC}]$ and to combine $[Z_{AC}]$ with $[Z_B]$ to produce the $Z$-matrix representation of the coupled elements. Finally, the $Z$-matrix is transformed to $S$-parameters to yield coupling between components $A$ and $B$ and the effect of coupling on any of these components.

A similar procedure can be used for evaluating coupling when we have a shielding conductor on the top but no side walls as described in Figure 10.4. The relevant expressions for this case are derived in Appendix G.

10.3 Computational Details for Calculation of Spurious Coupling in an Enclosure

Computational details for calculations of spurious coupling in free space given in Chapters 5 and 7 holds also for calculation of spurious coupling in an enclosure. However, to improve the accuracy of spurious coupling calculation in an enclosure we have to determine how many modes are needed to ensure
convergence of spurious coupling results.

10.3.1 Convergence of Coupling Results as a Function of the Number of Modes in the Enclosure

The accuracy and efficiency of spurious coupling calculation in an enclosure are a function of the number of modes taken into account in the calculation of the magnetic fields in an enclosure.

To reduce computation time needed to evaluate spurious coupling in rectangular enclosure, the number of modes taken into account in the calculation may be determined by checking the convergence of coupling results as a function of the number of modes considered in the calculation.

To determine the number of modes needed to ensure convergence of
coupling results in a rectangular enclosure, two examples are described in this section. The first example is coupling between two open ends in a rectangular enclosure as a function of frequency. The second example is coupling between two corners of a right-angled bend discontinuity in an enclosure as a function of spacing between the bends.

10.3.2 Convergence of Coupling Results between Two Open Ends in an Enclosure

The proposed method is used to analyze a 15mm gap in a 51Ω microstrip line on a substrate with relative permittivity of ε_r = 10.5(1 − j0.0023) and 1.27 mm thick, enclosed in a metal package with dimensions A = 3cm, B = 4.8cm, and H = 1.27cm as shown in Figure 10.5. S_{12} results were calculated for frequencies ranging from 3.5 GHz to 8.5 GHz for different numbers of modes. S_{12} results are listed in Table 10.1.

The results in Table 10.1 indicate that for coupling values higher than -40 dB, ten modes in each direction (m = n = p = 10) are needed to ensure good convergence of S_{12} results. For coupling values lower than -40 dB 20 × 20 × 20 modes are needed to obtain accurate results.

10.3.3 Convergence of Coupling Results between Two Right-Angled Bends in an Enclosure

The method described in Section 10.2 is used to evaluate spurious coupling at 20 GHz between 50Ω right-angled bends on a substrate with ε_r = 2.2, 10 mil thick substrate enclosed in a metal package with dimensions A = B = 4cm and H = 1cm, as shown in Figure 10.7. S_{13} results as a function of spacing between the right-angled bends for different numbers of modes are listed in Table 10.2.
Figure 10.5: Comparison of $S_{12}$ results between two open ends with Em program results and moment method results, $Y = 24 \text{mm}$.  

\[ \begin{align*} 
E_r &= 10.5 \\
h &= 1.27 \text{mm} \\
H &= 1.27 \text{cm} \\
A &= 3 \text{cm} \\
B &= 4.8 \text{cm} \\
Y &= 24 \text{mm} 
\end{align*} \]
Table 10.1: $S_{12}$ results between two open ends in an enclosure, as shown in Figure 10.5, for different numbers of modes.

<table>
<thead>
<tr>
<th>Frequency (GHz)</th>
<th>$S_{12}$ (dB) $m = n = p = 2$</th>
<th>$S_{12}$ (dB) $m = n = p = 6$</th>
<th>$S_{12}$ (dB) $m = n = p = 10$</th>
<th>$S_{12}$ (dB) $m = n = p = 16$</th>
<th>$S_{12}$ (dB) $m = n = p = 20$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>-59.11</td>
<td>-58.00</td>
<td>-55.75</td>
<td>-53.86</td>
<td>-53.66</td>
</tr>
<tr>
<td>5</td>
<td>-34.53</td>
<td>-35.88</td>
<td>-36.13</td>
<td>-36.41</td>
<td>-36.44</td>
</tr>
<tr>
<td>5.5</td>
<td>-16.91</td>
<td>-17.08</td>
<td>-17.11</td>
<td>-17.15</td>
<td>-17.10</td>
</tr>
<tr>
<td>7</td>
<td>-31.15</td>
<td>-30.36</td>
<td>-30.79</td>
<td>-29.99</td>
<td>-29.97</td>
</tr>
<tr>
<td>7.5</td>
<td>-31.73</td>
<td>-30.98</td>
<td>-30.79</td>
<td>-30.56</td>
<td>-30.54</td>
</tr>
<tr>
<td>8.5</td>
<td>-33.08</td>
<td>-32.70</td>
<td>-32.43</td>
<td>-32.12</td>
<td>-32.09</td>
</tr>
</tbody>
</table>

Table 10.2: $S_{13}$ results between two right-angled bends in an enclosure, as shown in Figure 10.7, for different numbers of modes.

<table>
<thead>
<tr>
<th>Spacing (mm)</th>
<th>$S_{13}$ (dB) $m = n = p = 10$</th>
<th>$S_{13}$ (dB) $m = n = p = 20$</th>
<th>$S_{13}$ (dB) $m = n = p = 30$</th>
<th>$S_{13}$ (dB) $m = n = p = 36$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>-65.68</td>
<td>-59.06</td>
<td>-58.62</td>
<td>-58.01</td>
</tr>
<tr>
<td>3</td>
<td>-65.62</td>
<td>-69.59</td>
<td>-66.95</td>
<td>-65.90</td>
</tr>
<tr>
<td>4</td>
<td>-55.96</td>
<td>-55.79</td>
<td>-56.53</td>
<td>-57.10</td>
</tr>
<tr>
<td>5</td>
<td>-51.41</td>
<td>-50.92</td>
<td>-51.57</td>
<td>-51.28</td>
</tr>
</tbody>
</table>

The results in Table 10.2 indicate that at least $30 \times 30 \times 30$ modes are needed to ensure good convergence of coupling results, $S_{13}$, between two corners of right-angled bends in a metal package with dimensions $A = B = 4cm$ and $H = 1cm$. 
10.4 Examples of Parasitic Coupling Evaluation in the Presence of a Rectangular Enclosure

When a microwave circuit is enclosed in a metal package electrically large enough to support resonant modes which modify the coupling between various parts of the circuit, the circuit’s performance is changed considerably. Package effects are investigated in this section by evaluating coupling between two open ends in a metal enclosure and by evaluating coupling between two corners of a 50Ω right-angled bend in a metal enclosure.

10.4.1 Coupling between Two Colinear Open Ends in a Metal Enclosure

The coupling between two colinear open ends in a 51Ω microstrip line on $\varepsilon_r = 10.5$ substrate (1.27mm thick) has been investigated as a function of frequency and enclosure dimensions. The proposed method is used to estimate coupling between two open ends located 15mm apart in a metallic package with dimensions $A = 3cm$, $B = 4.8cm$ and $H = 1.27cm$. The results are compared to Em program [54] results, Em is a fullwave three-dimensional analysis. This analytical fullwave numerical approach invokes a Galerkin method of moments technique which develops the dyadic Green’s function operator as a bi-dimensional infinite vectorial summation of homogeneous waveguide eigenfunctions. $S_{12}$ results are compared also to moment method results [50] when a low-loss dielectric substrate coated with a resistive film was placed on the cover to damp the resonant modes. $S_{21}$ results for frequencies ranging from 5 GHz to 8.5 GHz when the microstrip lines are centered at $Y = 24mm$ are plotted in Figure 10.5. $S_{21}$ results in a metallic package calculated using the proposed method and modal expansion in cavities are plotted as curve 2. $S_{21}$ results in free space, using MNM, are plotted as curve 1. One may note that $S_{21}$ results in an open config-
uration, with no enclosure, are very low. At 8 GHz the transmission value $S_{21}$ is
-63.44 dB. $S_{21}$ results using the Em program are plotted as curve 3. $S_{21}$ results,
when a low loss dielectric substrate coated with a resistive film was placed on
the cover, given in [50] are plotted as curve 4. In the frequency range of 5 GHz
to 8.5 GHz the $TM_{110}$ and the $TM_{120}$ modes are resonant at 5.6 GHz and at 7.5
GHz respectively in this enclosure. Maximum power will couple to the $TM_{110}$
mode by placing the circuit at $Y = 24\text{mm} \ (\approx \frac{1}{2})$ and no power will couple to the
$TM_{120}$ mode at this location. The three sets of results support this prediction.
$S_{21}$ results at 5.6 GHz using MNM and the Em program are -3.0 dB and -1.5
dB respectively. However, the moment method $S_{21}$ result at 5.6 GHz is -7.5 dB.
One may conclude that the resistive film is damping the resonant mode by 4.5
to 6 dB approximately.

$S_{21}$ results when the circuit is placed at $Y = 12\text{mm}$ are plotted in Figure
10.6. $S_{21}$ results in an enclosure using the proposed method are plotted as curve
2. $S_{21}$ in free space using MNM are plotted as curve 1. $S_{21}$ results using the
Em program are plotted as curve 3. $S_{21}$ results using moment method [50] are
plotted as curve 4. We may note that the three sets of results show resonances
at 5.6 GHz and at 7.5 GHz.

10.4.2 Parasitic Coupling between Two Corners of Right-Angled
Bends as a Function of Enclosure Dimensions

The effect of enclosing two 50Ω right-angled bends on $\varepsilon_r = 2.2$ substrate,
10 mil thick, is reported in this section as a function of enclosure dimensions
for an enclosure with six metallic walls. Results in this section present values of
spurious coupling when interaction among the equivalent magnetic currents at
the outer corners of the bends are taken into account. Values of the transmission
coefficient $S_{13}$ in dB at 20 GHz for spacing values ranging from two millimeters
Figure 10.6: Comparison of $S_{12}$ results between two open ends with EM program results and moment method results, $Y = 12\,\text{mm}$.

to five millimeters are plotted in Figure 10.7. $S_{13}$ results without any enclosure are given as curve 1 of this figure. $S_{13}$ results in a closed enclosure, six metallic walls, with dimensions $A = B = 4\,\text{cm}$ and $H = 1\,\text{cm}$ are plotted as curve 2. $S_{13}$ results in a closed enclosure with dimensions $A = B = 6\,\text{cm}$ and $H = 1\,\text{cm}$ are plotted as curve 3. By comparison of curves 2 and 3 in Figure 10.7 one observes that spurious coupling is considerably higher in an electrically larger package. As an example, $S_{13}$ results for an enclosure with $A = B = 6\,\text{cm}$ and $H = 1\,\text{cm}$ for two millimeters spacing between the right-angled bends is higher by 6.67 dB than $S_{13}$ results in an enclosure with $A = B = 4\,\text{cm}$ and $H = 1\,\text{cm}$, for the same spacing. These results support the assumption that spurious coupling is enhanced in electrically large enclosures. Curve 2 in Figure 10.7 indicates that as the circuit
is moved nearer to the side walls, spurious electromagnetic coupling increases. As an example, $S_{13}$ results for an enclosure with $A = B = 4cm$ and $H = 1cm$, for 5 millimeters spacing between the two right-angled bends is higher by 14.57 dB than $S_{13}$ results in free space for the same spacing.

Values of the transmission coefficient $S_{13}$ in dB at 20 GHz between two corners of 50Ω right-angled bends on $\varepsilon_r = 2.2$ substrate, 10 mil thick, for spacing values ranging from two millimeters to five millimeters are plotted in Figure 10.8. $S_{13}$ results without any enclosure are plotted as curve 1. $S_{13}$ results in a closed enclosure, six metallic walls, with dimensions $A = B = 4cm$ and $H = 3cm$ are plotted as curve 2. $S_{13}$ results in a closed enclosure with dimensions $A = B = 6cm$ and $H = 3cm$ are plotted as curve 3. One may note that $S_{13}$ results in an enclosure with dimensions $A = B = 4 cm$ and $H = 3 cm$, curve 2, converges to $S_{13}$ results in free space, curve 1, for spacing values from two millimeters to four millimeters. However, as the circuit is moved nearer to the side walls, spacing of five millimeters, the difference in $S_{13}$ result in an enclosure, curve 2, and $S_{13}$ result in free space, curve 1, is 2.92 dB. $S_{13}$ results in an enclosure with dimensions $A = B = 6cm$ and $H = 3cm$, for spacing values of 4mm and 5mm, are higher than $S_{13}$ results in an enclosure with $A = B = 4cm$ and $H = 3cm$ by 4.63 dB and 7.95 dB respectively. These results support the assumption that spurious coupling increases in electrically large enclosures.
Curve 1: $S_{13}$ results in free space.

Curve 2: $S_{13}$ results in an enclosure with $A = B = 4 \text{ cm}$, $H = 1 \text{ cm}$. 

Curve 3: $S_{13}$ results in an enclosure with $A = B = 6 \text{ cm}$, $H = 1 \text{ cm}$. 

Figure 10.7: $S_{13}$ results between two right-angled bends in an enclosure, $H = 1 \text{ cm}$. 
Figure 10.8: $S_{13}$ results between two right-angled bends in an enclosure with $H = 3\text{cm}$. 

Curve 1: No enclosure.

Curve 2: $A = B = 4\text{ cm}$, $H = 3\text{ cm}$.

Curve 3: $A = B = 6\text{ cm}$, $H = 3\text{ cm}$. 

$F = 20\text{ GHz}$  
$Z_0 = 50\text{ Ohm}$
CHAPTER XI

CONCLUDING REMARKS

11 Conclusions

The salient features of the research efforts described in this thesis are summarized in this chapter. Suggestions for possible improvements in the multiport network model for evaluation of radiation loss and spurious coupling between microstrip discontinuities are discussed. Possible applications of the method developed and other future extensions are also included.

11.1 Summary of Salient Results

11.1.1 Multiport Network Model for Evaluating Radiation Loss from Microstrip Discontinuities

A convenient and versatile method (based on a multiport modeling approach) for evaluating radiation loss associated with discontinuities in microstrip circuits has been developed in this thesis. A planar multiport network model of the discontinuity configuration and the segmentation method are used to evaluate voltage distribution around the edges of the discontinuity. This voltage distribution is expressed as an equivalent magnetic current line source distribution which is used to calculate the far-zone field for radiation loss calculations. The radiation loss may be incorporated in the S-parameters representation of the discontinuities by combining the Z-matrix of the multiport model of...
the discontinuity with the impedance matrix for an equivalent edge impedance network. Results for power radiation from a gap discontinuity, from chamfered bends and a 20Ω open-end stub that have not been previously available in literature are given in this thesis. This approach is suitable for being included in microwave CAD packages.

Radiation Q-factors obtained from measurements in resonators incorporating right-angle bends show a fair agreement with estimated values. This provides an experimental verification of the proposed method. The computed results are in good agreement with the Poynting vector method applicable for simple geometries.

11.1.2 Multiport Network Model for Evaluating Spurious Coupling between Discontinuities in Microstrip Circuits

The spurious coupling between two discontinuities (due to external fields) may be incorporated in the multiport network model by connecting a mutual coupling network between the two discontinuities. As before, the multiport network model is used to find voltage distribution around discontinuity edges. In this case equivalent magnetic currents are used to compute the electric current induced at the location of the other discontinuity. This yields a mutual admittance matrix representing the spurious coupling.

The results presented in Chapters 6 and 7 show that spurious coupling between discontinuities cannot be neglected and can have significant effect as in the case of the transmission coefficient of the double-stub circuit as described in Chapter 6.

The method of computation of spurious coupling reported in this thesis is expected to be a very useful tool for studying the effect of spurious coupling on the performance of high-density MMIC's. This will help in formulating the
design rules so that the undesirable coupling effects are avoided. This approach is suitable to be included in microwave CAD packages.

Two different experimental verifications of the mutual coupling network model are reported in Chapter 6. In case of the coupling among two lines (each incorporating two step discontinuities), good agreement between measured results and computed results using the MCN model verifies the proposed method. The second experimental verification is obtained by comparing calculated results using MCN model with calculated results using fullwave analysis and with the experimental results on a double-stub circuit (measurements obtained from Texas Instruments Microwave Laboratory).

11.1.3 Planar-Lumped Model for Coupled Microstrip Line Discontinuities

A novel modeling approach for characterization of coupled microstrip junction effects has been proposed in Chapter 8. This approach is an extension of the planar waveguide model used extensively for characterization of single microstrip line discontinuities.

For coupled lines, fields underneath the two strips and those fringing at the outer edges are modeled by equivalent planar waveguides. Electric and magnetic field couplings in the gap region are modeled by a lumped network. The proposed model has been verified by comparing results for a coupled line section with those obtained from conventional coupled line analysis. The proposed model is shown to be valid for higher-order modes also. The cutoff frequencies for higher-order modes predicted by this model agree very well with those obtained from fullwave spectral domain analysis of coupled microstrip lines.

This modeling approach is applied to a coupled microstrip section with
chamfered right-angled bends connected to single microstrip lines and results in good agreement with experimental values. This good agreement provides a verification for the proposed model. The proposed model has been applied to a number of different coupled line structures. These include a one-section coupler with abrupt 90° bend connections to single microstrip lines and also with chamfered bends at output ports. The proposed model has also been applied to the analysis of step discontinuities between coupled line sections in a three-section directional coupler. In all these cases, discontinuities are seen to cause appreciable change in performance.

The planar-lumped model has been extended to investigate radiation and spurious coupling between coupled microstrip discontinuities in the same manner as the planar waveguide model approach has been extended, as described in Chapters Two and Five, for study of radiation and parasitic coupling between microstrip discontinuities.

11.1.4 The Effect of Enclosures on Parasitic Coupling between Microstrip discontinuities

MMICs are often enclosed in metallic enclosures to protect the circuit from the environment and to provide electrical isolation between different parts of the subsystem. Enclosures support resonant modes which may influence external coupling among various parts of the circuit and modify the circuit performance considerably. Modal expansion of fields in enclosure cavity and the multiport network model are used to evaluate spurious coupling between discontinuities in a rectangular enclosure. Numerical results given in Chapter 10 show that electrically large enclosures may support resonant modes with resonances lying within the frequency of operation of the enclosed circuit and coupling between the circuit and these resonant modes cannot be neglected.
11.2 Suggestions for Further Work

11.2.1 Incorporating Radiation Loss and Spurious Coupling among Discontinuities in Microstrip Antenna Arrays

In microstrip antenna arrays, relatively thicker substrates with low dielectric constants are used and the feed network is usually printed on the same substrate. Discontinuities in the feed network can result in substantial spurious coupling and radiation loss. The major effects of this radiation and spurious coupling phenomena are: additional signal loss in the feed network, higher side lobe levels and beam squint.

The multiport network model for evaluating radiation loss and spurious coupling among microstrip discontinuities may be used to include these effects in the analysis of microstrip antenna arrays. This is achieved by calculating radiation loss from the feed network and adding MCN between the discontinuities.

This procedure will improve the accuracy of design and analysis of microstrip arrays and reduce the number of iterations needed to design microstrip arrays.

11.2.2 Evaluation of Radiation Loss and Spurious Coupling among Discontinuities in Microstrip Circuits Covered with a Dielectric Layer

In many antenna system applications, microstrip circuits and antenna arrays require a dielectric cover layer to provide protection from physical damage and the environment. The presence of a dielectric layer can seriously alter the performances of the antenna and associated feed network (e.g. change in resonance frequency, S-parameters) and may cause excitation of surface waves.

Microstrip transmission lines with a cover layer have been investigated
in [55, 56]. A useful formula relating the effective dielectric constant to the line parameters is given in [55]. Spectral domain analysis for microstrip lines discussed in Appendix C may be used to calculate the effective dielectric constant, characteristic impedance and the effective width of a microstrip line with a cover layer. These microstrip line parameters may be used to evaluate radiation loss and spurious coupling among discontinuities in microstrip circuits covered with a dielectric layer using the MNM and the planar-lumped model developed in this research. The effects of surface waves, which are excited because of the presence of the cover layer, need to be incorporated in the analysis.

11.2.3 Improved MNM for Evaluation of Spurious Coupling in an Enclosure

The MNM for evaluation of spurious coupling in an enclosure may be improved by using strips of magnetic current in place of line sources, by incorporating substrate effects and resistive layers in the analysis.

Use of Strips of Magnetic Current in Place of Line Sources. As described in Section 5.2, the external radiation field produced by discontinuities is evaluated by modeling the fringing field at the microstrip edges by equivalent magnetic current line sources. These equivalent magnetic current line sources can be placed at the physical edges of the microstrip structure or at the edges of the configuration with effective dimensions. Either of these approximations is fairly good for calculations of the far-zone field. However, for evaluation of the spurious coupling, we need to calculate fields at much shorter distances and it is desirable to have a more accurate model for edge fields. The improved model for edge fields employs a two-dimensional distribution of the equivalent magnetic current density in place of line current sources, as described in Section 5.2.

Spurious coupling results given in Chapter 10 employed line current
sources to reduce cpu-time. More accurate results may be obtained by using strips of magnetic current in place of line sources.

**Incorporating Resistive Layers in the Analysis.** To reduce coupling of power to resonant modes in an enclosure, two methods are usually used. The first method is to fix a microwave absorbing layer to the metal walls. In the second method the resonant cavity modes may be damped by placing a dielectric substrate coated with resistive film in the cavity. The algorithm proposed in Chapter 10 may be extended to evaluate spurious coupling between discontinuities in an enclosure when absorbing layers or resistive films on dielectric layers are used to reduce package effects. This may be done by finding fields produced by a magnetic current source in an enclosure with more than two homogeneous regions. This problem may be solved by finding solutions in each region such that tangential components of $E$ and $H$ are continuous across the common boundary.
REFERENCES


APPENDIX A

GREEN FUNCTION APPROACH FOR COMPUTATION OF Z-MATRICES OF PLANAR SEGMENTS

The $Z$-matrix of a two-dimensional planar microstrip segment can be calculated using the Green's function approach. The method may be applied for shapes for which the wave equation can be solved by using separation of variables technique. Expression for the Green's function for a rectangular segment is given in [A.3].

A.1 $Z$-Matrix of Planar Network with Magnetic Boundary

When a planar microstrip component, as shown in Figure A.1, is excited by a $z$-directed current density $J_Z$ at any arbitrary point $(x_0, y_0)$ inside the segment, the wave equation (in rectangular coordinate system) is written as [13]

$$\left( \nabla^2 + k^2 \right) V(x, y) = i \omega \mu h J_z(x_0, y_0)$$  \hspace{1cm} (A.1)

where

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$  \hspace{1cm} (A.2)

and $h$ is the substrate height and

$$k = \omega \sqrt{\mu \varepsilon}$$  \hspace{1cm} (A.3)

The boundary condition at magnetic walls along the segment periphery is given
Figure A.1: A microstrip component excited by a $z$-directed electric current source at $(X_0, Y_0)$

by

$$\frac{\partial V}{\partial n} = 0 \quad (A.4)$$

The solution of equation (A.1) in terms of the Green's function for an arbitrary source distribution inside the component may be written as

$$v(x, y) = \int \int_A G(x, y | x_0, y_0) J_z(x_0, y_0) \, dx_0 \, dy_0 \quad (A.5)$$

where $A$ denotes the area of the component enclosed by the magnetic wall.

The Green's function $G$ is the solution of the wave equation for delta function excitation

$$(\nabla^2 + k^2) G(x, y | x_0, y_0) = -i\omega \mu \epsilon \delta(x - x_0) \delta(y - y_0) \quad (A.6)$$
The boundary condition for \( G \) is the same as the boundary condition for the voltage \( V \). When the planar segment is excited at the periphery, the boundary condition for the voltage at a coupling port is

\[
\frac{\partial V}{\partial n} = i \omega \mu h \ J_z
\]  

(A.7)

The relation between the voltage along the periphery and the Green's function may be written as

\[
V(s) = \int_C G(S \mid S_0) \ J_z(S_0) \ dS_0 \tag{A.8}
\]

where \( C \) is the integration contour along the periphery and \( S, S_0 \) are the distances measured along the periphery \( C \). The integration in (A.8) is along the coupling ports only, since \( J_z = 0 \) elsewhere. For \( n \) coupling ports of width \( W_j \) along the periphery \( V(S) \) may be written as:

\[
V(s) = \sum_{j=1}^{n} \int_{W_j} G(S/S_0) \ J_{zj}(S_0) \ dS_0 \tag{A.9}
\]

The relation between the current at the \( j \)-th port and the current density \( J_{zj}(S_0) \) is given as

\[
I_j = \int_{W_j} J_{zj}(S_0) \ dS_0 \tag{A.10}
\]

The current density is assumed to be uniform along ports with small widths. For ports with small widths, the current \( I_j \) is written as

\[
I_j = W_j \ J_{zj}(S_0) \tag{A.11}
\]

A voltage \( V_i \) at the \( i \)-th coupling port is defined as the average voltage along the port width. The voltage \( V_i \) may be written as

\[
V_i = \sum_j \frac{I_j}{W_i W_j} \int_{W_i} \int_{W_j} G(S/S_0) \ dS_0 \ dS \tag{A.12}
\]
The voltage at port \( i \) is related to the current at port \( j \) by the \( Z_{ij} \) element of the \( Z \)-matrix

\[
Z_{ij} = \frac{1}{W_i W_j} \int_{W_i} \int_{W_j} G(S/S_0) \, dS_0 \, dS
\]  
(A.13)

### A.2 Computation of Green’s Function

The Green’s function for a planar segment may be computed by representing it as a summation of eigenfunctions \( \psi_m \) which satisfy the wave equation

\[
\nabla^2 \psi_m + k_m^2 \psi_m = 0
\]  
(A.14)

\( \psi_m \) satisfies the boundary condition \( \frac{\partial \psi_m}{\partial n} = 0 \) at the periphery. \( k_m \) is the eigenvalue corresponding to the eigenfunction \( \psi_m \). The set \( \psi_m \) is an orthonormal set with

\[
\int \int_A \psi_m(x) \psi_n^*(x) \, dx \, dy = \begin{cases} 1 & \text{if } n = m \\ 0 & \text{otherwise} \end{cases}
\]  
(A.15)

and the Green’s function may be expanded as series of \( \psi_m \)

\[
G(x, y | x_0, y_0) = \sum_{m} D_m(x_0, y_0) \psi_m(x, y)
\]  
(A.16)

Inserting (A.16) into (A.6) and using equation (A.15), the coefficients \( D_m \) may be written as

\[
D_m = \frac{i\omega \mu \epsilon \psi_m(x_0, y_0)}{k_m^2 - k^2}
\]  
(A.17)

and the Green’s function is written as

\[
G(x, y | x_0, y_0) = i\omega \mu \epsilon \sum_{m=0}^{\infty} \frac{\psi_m(x, y) \psi_m^*(x_0, y_0)}{k_m^2 - k^2}
\]  
(A.18)

### A.3 Green’s function for a rectangular planar segment

The Green’s function for a rectangular microstrip segment may be written as [13]

\[
G(x, y | x_0, y_0) = \frac{i\omega \mu \epsilon}{ab} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \sigma_m \sigma_n \cos(k_x x_0) \cos(k_y y_0) \cos(k_x x) \cos(k_y y) \frac{1}{k_x^2 + k_y^2 - k_m^2}
\]  
(A.19)
where

\[ k_x = \frac{m\pi}{a}, \]
\[ k_y = \frac{n\pi}{b} \]

and

\[ \sigma_m = \begin{cases} 1 & \text{if } m = 0 \\ 2 & \text{otherwise} \end{cases} \]

\[ k^2 = \omega^2 \mu_0 \varepsilon_0 \varepsilon_{re} \]

\( \varepsilon_{re} \) is the complex dielectric constant \( \varepsilon_{re} = \varepsilon_r(1 - j\delta) \). \( \delta \) is the loss tangent of the dielectric substrate.
APPENDIX B

SEGMENTATION METHOD

When two multiport networks $A$ and $B$ are connected, their impedance matrices can be combined using the segmentation technique to obtain the impedance matrix of the resulting network "C" [13]. The two segments are connected together at $n$ interconnected ports. Let $q_1$ to $q_n$ be interconnected ports for segment $A$ and $r_1$ to $r_n$ be the interconnected ports for segment $B$. The computational effort is reduced if ports are numbered such that $q_1$ is connected to $r_1$, $q_2$ to $r_2$ . . . etc [57]. The external ports $P_a$ for segment $A$ and $P_b$ for segment $B$ are numbered before the interconnected ports. The voltages are related to the currents by the following equation:

$$
\begin{bmatrix}
V_p \\
V_q \\
V_r
\end{bmatrix} =
\begin{bmatrix}
Z_{pp} & Z_{pq} & Z_{pr} \\
Z_{qp} & Z_{qq} & Z_{qr} \\
Z_{rp} & Z_{rq} & Z_{rr}
\end{bmatrix}
\begin{bmatrix}
I_p \\
I_q \\
I_r
\end{bmatrix}
$$

(B.1)

where $Z_{pp}$, $Z_{pq}$ etc. are submatrices of appropriate dimensions. $V_p$ and $I_p$ are vectors representing voltages and currents at the external ports. $V_q$ and $V_r$ are voltage vectors at the interconnected ports. $I_q$ and $I_r$ are current vectors at the interconnected ports.

The conditions for connecting the ports $q_i$ and $r_i$ are obtained from Kirchoff’s circuit law. The voltages at two connected ports are equal and the sum of currents at two connected ports is zero.

$$V_q = V_r$$

(B.2.a)
\[ I_q + I_r = 0 \]  \hspace{1cm} (B.2b)

Applying Kirchoff's circuit law equations B.1 and B.2 and eliminating \( V_q \) and \( I_r \) we obtain the \( Z \)-matrix \( Z_p \) for the overall network.

\[ Z_p = Z_{pp} + (Z_{pq} - Z_{pr}) (Z_{qq} - Z_{qr} - Z_{rr} + Z_{rr})^{-1} (Z_{rp} - Z_{qp}) \]  \hspace{1cm} (B.3)

In most cases \( q \) and \( r \) ports belong to two physically separate segments and \( Z_{qr} = Z_{rq} = 0 \). Equation B.3 can be written as:

\[ Z_p = Z_{pp} + (Z_{pq} - Z_{pr}) (Z_{qq} + Z_{rr})^{-1} (Z_{rp} - Z_{qp}) \]  \hspace{1cm} (B.4)

The voltages \( V \) at the connected ports \( (q = r) \) are related to the current \( I_p \) at the external ports by

\[ V = [Z_{qp} + (Z_{qq} - Z_{qr}) (Z_{qq} + Z_{rr})^{-1} (Z_{rp} - Z_{qp})] I_p \]  \hspace{1cm} (B.5)
APPENDIX C

SPECTRAL DOMAIN ANALYSIS FOR EVALUATION OF MICROSTRIP LINE PARAMETERS

6.1 Introduction

The spectral domain method has proven to be a powerful tool for the analysis of microstrip lines and antennas [58]. It is a fullwave analysis that can easily accommodate multi-layered structures and account for factors such as dielectric loss and edge effect [23].

The Fourier transform representation of electromagnetic fields and sources facilitates the replacement of differential or integral spatial domain field equations by algebraic equations in the spectral domain. This results in a substantial simplification of the relationship between fields and their sources. Both differential wave equations and integral equations are transformed into single algebraic equations in which the spectral field is the product of the spectral domain source and the spectral domain Green's function.

The spectral domain formulation for microstrip line and for coupled microstrip line is given in [23] [59].

In [23] the formulation for evaluation of the characteristic impedance of a single microstrip line and a coupled microstrip line is not given. In both [23] and [59] there are not enough details about the numerical solution of the problem. In this appendix, an accurate and efficient numerical solution for
single microstrip lines is presented. Spectral domain solution for coupled lines is developed in Appendix D.

C.2 Formulation of the Problem

Referring to the system of coordinates shown in Figure C.1, the top surface of the substrate is taken to be $y = 0$

![Figure C.1: Open microstrip resonant structure.](image)

The integral equations for current components $J_x$ and $J_z$ in space domain are given by

$$\int \int [Z_{xx}(x-x', z-z') J_x(x', z') + Z_{xz}(x-x', z-z') J_z(x', z')] dx'dz' = 0 \quad (C.1)$$

and

$$\int \int [Z_{xz}(x-x', z-z') J_x(x', z') + Z_{zz}(x-x', z-z') J_z(x', z')] dx'dz' = 0 \quad (C.2)$$
where the integrations are over the strip and the equations hold for \((x, z)\) on the strip. The transverse electric fields \(E_x\) and \(E_y\) are zero on the strip. Equations (C.1) and (C.2) are transformed to the spectral domain and we obtain
\[
\begin{bmatrix}
\tilde{E}_x (\alpha, \beta) \\
\tilde{E}_z (\alpha, \beta)
\end{bmatrix} =
\begin{bmatrix}
\tilde{Z}_{xx}(\alpha, \beta) & \tilde{Z}_{xz}(\alpha, \beta) \\
\tilde{Z}_{zx}(\alpha, \beta) & \tilde{Z}_{zz}(\alpha, \beta)
\end{bmatrix}
\begin{bmatrix}
\tilde{J}_x (\alpha, \beta) \\
\tilde{J}_z (\alpha, \beta)
\end{bmatrix}
\]  
(C.3)
The tangential spectral electric fields are a product of the spectral dyadic Greens's function and the tangential currents, where the Fourier transform is defined as
\[
\hat{\phi}(\alpha, \beta) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \phi(x, z) \, e^{j(ax+\beta z)} \, dx \, dz
\]  
(C.4)
and the inverse Fourier transform is defined as
\[
\phi(x, z) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \hat{\phi}(\alpha, \beta) \, e^{-j(ax+\beta z)} \, d\alpha \, d\beta
\]  
(C.5)
The spectral domain Green's dyad for a grounded dielectric slab is given in [58, 59]. Detailed derivation for a spectral domain Green's dyad for a grounded multi-layered dielectric slab is given in [60]. The grounded multi-layered dielectric slab may have continuous as well as discontinuous variations in the \(y\) direction and uniform in the transverse \((x, z)\) directions. Thus it lends itself to transmission line analogy as shown in Figure C.2. \(Z_0^e\) and \(Z_0^h\) are input impedances looking into the equivalent circuits at \(y = 0\) and may be expressed as
\[
Z_0^e = \frac{1}{Y_+^e + Y_-^e} \quad \text{TM modes} 
\]  
(C.6.a)
\[
Z_0^h = \frac{1}{Y_+^h + Y_-^h} \quad \text{TE modes} 
\]  
(C.6.b)
\[
Y_+^e = Y_{TM_1} \coth \gamma_1 h 
\]  
(C.7.a)
\[
Y_-^e = Y_{TM_2} 
\]  
(C.7.b)
\[
Y_+^h = Y_{TE_1} \coth \gamma_1 h 
\]  
(C.8.a)
\[
Y_-^h = Y_{TE_2} 
\]  
(C.8.b)
where

\[ Y_{TM} = \frac{j \omega \varepsilon_0 \varepsilon_i}{\gamma_i} \]  

(3.9)

where

\[ \gamma_i^2 = \alpha^2 + \beta^2 - \varepsilon_i \mu_0 \omega_0^2 \]  

(C.10)

and where

\[ Y_{TE} = \frac{\gamma_i}{j \omega \mu} \]  

(C.11)

Figure C.2: Spectral domain equivalent circuit.

The spectral domain dyadic Green's function may be written as

\[ \hat{Z}_{xx} (\alpha, \beta) = N_x^2 \hat{Z}_0^x + N_x^2 \hat{Z}_0^b \]  

(C.12)

\[ \hat{Z}_{zz} (\alpha, \beta) = N_z N_z (\hat{Z}_0^z + \hat{Z}_0^b) \]  

(C.13)

\[ \hat{Z}_{zz} (\alpha, \beta) = N_z^2 \hat{Z}_0^z + N_z^2 \hat{Z}_0^b \]  

(C.14)

where

\[ N_x = \frac{\alpha}{\sqrt{\alpha^2 + \beta^2}} \]

\[ N_z = \frac{\beta}{\sqrt{\alpha^2 + \beta^2}} \]  

(C.15)
The unknown spectral currents \( \tilde{J}_x \) and \( \tilde{J}_z \) are expanded in terms of linear combinations of known basis functions

\[
\tilde{J}_x (\alpha, \beta) = \sum_{m=1}^{M} c_m \tilde{J}_{x_m} (\alpha, \beta)
\]

\[
\tilde{J}_z (\alpha, \beta) = \sum_{n=1}^{N} d_n \tilde{J}_{z_n} (\alpha, \beta)
\]

(B.16)

Basis functions for microstrip line calculation are given in [23].

If we substitute equations (C.16) into (C.3) and take the inner product of the resulting equation with each of the expansion functions, the result is a homogeneous matrix equation:

\[
\sum_{m=1}^{M} k_{pm}(zz) c_m + \sum_{n=1}^{N} k_{pn}(zz) d_n = 0 \quad p = 1, 2, \ldots, M
\]

\[
\sum_{m=1}^{M} k_{qm}(zz) c_m + \sum_{n=1}^{N} k_{qn}(zz) d_n = 0 \quad q = 1, 2, \ldots, N
\]

(C.17)

where the typical matrix element is given by

\[
k_{pn}(zz) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \tilde{J}_{zp}(\alpha, \beta) \tilde{z}_{zz}(\alpha, \beta) \tilde{J}_{zn}(\alpha, \beta) \, d\alpha d\beta
\]

(C.18)

Since the \( k_{pm} \)'s are functions of frequency, a non-trivial solution of (C.17) is obtained using a complex root-searching routine that searches for the complex frequency that makes the determinant of the coefficient of (C.17) zero. The complex constants \( (c_n, d_n) \) specify the current distribution on the strip. The right-hand side of (C.17) is zero since \( J_z \) is zero outside the strip and the tangential electric fields are zero on the strip.

For microstrip lines, equation (C.18) can be written as

\[
k_p(x) = \int_{-\infty}^{\infty} \tilde{J}_{zp}(\alpha, \beta) \tilde{G}_{zz}(\alpha, \beta) \tilde{J}_z(\alpha, \beta) \, d\alpha
\]

(C.19)
For microstrip lines the $k_p$'s coefficients are functions of the propagation constant $\beta$ in the microstrip line. A non-trivial solution of (C.17) is obtained using a complex root-searching routine that searches for the complex propagation constant that makes the determinant of the coefficient matrix (C.17) zero. This yields the value of the propagation constant $\beta$.

The effective dielectric constant is obtained using the following equation:

$$\varepsilon_{re}(f) = \frac{\beta^2}{k_0}$$

(C.20)

where $k_0$ is the wave number in free space.

C.3 Computations of Characteristic Impedance for Microstrip Lines

The spectral domain method may be used to evaluate the characteristic impedance of microstrip lines. There are a number of different possible definitions of the characteristic impedance of microstrip lines. As discussed in [61], the behavior and results for the characteristic impedance of microstrip lines depends on the definition used to calculate $Z_0$. The possible definition of the characteristic impedance for microstrip lines are

$$Z_1 = \frac{\tilde{V}}{T}$$

$$Z_2 = \frac{V_0}{T}$$

$$Z_3 = \frac{P}{II^*}$$

$$Z_4 = \frac{V_0 V_0^*}{P}$$

$$Z_5 = \frac{\tilde{V} \tilde{V}^*}{P}$$

(C.21)

where $\tilde{V}$ is the average voltage on the line, $V_0$ is the voltage at the line center and $P$ is the average power. It is shown in [61] that $Z_1$ decreases slightly with frequency, $Z_3$ decreases considerably as a function of frequency, $Z_3$ increases as
frequency tends toward infinity. In [61] it is shown that \( Z_3 \) decreases slightly at low frequencies and increases slightly for higher frequencies, \( Z_2 \) increases more rapidly with frequency and \( Z_6 \) increases considerably with frequency. As frequency tends toward infinity, \( Z_2, Z_3 \) and \( Z_4 \) results approach a single value.

In [23] \( Z_0 \) is defined, like \( Z_2 \), as a voltage per longitudinal current basis with the voltage \( V_0 \) equal to the line integral over the electric field from the strip center perpendicular to the ground. In this approach, the electric field is calculated in spectral domain and transformed to space domain to calculate \( V_0 \).

For results reported in this thesis, we used definition \( Z_3 \). An accurate expression for the characteristic impedance may be obtained in terms of the power transmitted and the current in the direction of propagation as shown in the following equations:

\[
Z_0 = \frac{2}{I_s^2} \frac{P_{\text{avg}}}{I_s^2} \quad \text{(C.22)}
\]

where

\[
P_{\text{avg}} = \frac{1}{2} R_e \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (E_x H_y^* - E_y H_x^*) \, dx \, dy \quad \text{(C.23)}
\]

and

\[
I_s = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} J_s \, dx \quad \text{(C.24)}
\]

where the electric and magnetic fields may be obtained using the inverse Fourier transform (C.5). However, Parseval’s theorem may be applied to (C.23) to obtain

\[
P_f = \frac{1}{4\pi^2} \int_{0}^{\infty} \int_{-\infty}^{\infty} (\tilde{E}_x \tilde{H}_y^* - \tilde{E}_y \tilde{H}_x^*) \, d\alpha \, dy \quad \text{(C.25)}
\]

where Parseval’s theorem is stated as

\[
\int f(x) \cdot g(x) \, dx = \frac{1}{2\pi} \int F(\alpha) \cdot G(\alpha) \, d\alpha \quad \text{(C.26)}
\]
The integration with respect to \( y \) is performed analytically and the integration with respect to \( \alpha \) is carried out numerically in each of the dielectric regions. It was found that applying the Simpson rule to solve the integration in (C.25) is not efficient and not accurate. Using the Simpson rule the integration results are a function of the step size and number of steps taken to perform the numerical integration. The integrand in (C.25) is a product of Bessel functions and exponential function, an oscillating function when \( \alpha \) goes to infinity or to large numbers. From this reasoning, applying the Simpson rule to solve (C.25) is not useful. However, using Gauss quadrature method or adaptive Romberg integration is very efficient and accurate. Use of Gauss quadrature integration and adaptive Romberg integration yields identical results.

The integrand in (C.25) causes numerical overflow problems when the variable of integration \( \alpha \) has values greater than a thousand, since the integrand has \( \coth(2\gamma_i h) \) term where \( \gamma_i = \alpha^2 + \beta^2 - \varepsilon_i \mu \omega_0^2 \)

\[
\coth(2\gamma_i h) = \frac{e^{2\gamma_i h} - e^{-2\gamma_i h}}{e^{2\gamma_i h} + e^{-2\gamma_i h}} \tag{C.27}
\]

If the overflow problem is avoided and the integration is performed for values of \( \alpha \) less than a thousand, results for \( Z_0 \) will not converge and will not be accurate. Moreover, the characteristic impedance will decrease as a function of frequency instead of increasing as reported in [23] and [61].

The overflow problem is solved by multiplying the numerator and denominator by \( e^{-\delta} \) (where \( \delta \) is a number between 20 and 40).

\[
\coth(2\gamma_i h) = \frac{e^{2\gamma_i h - \delta} - e^{-2\gamma_i h - \delta}}{e^{2\gamma_i h - \delta} + e^{-2\gamma_i h - \delta}} \tag{C.28}
\]

This technique allows pushing the limit of integration up to ten thousand. This procedure gives an opportunity to check convergence of results of the
characteristic impedance and effective dielectric constant for microstrip lines. This is very important at high frequencies since values of $\beta$ at high frequencies are higher and the ratio strip width versus wavelength increases. Therefore, the limit of integration needs to be higher for higher frequencies. Numerical results obtained using this algorithm indicate that the effective dielectric constant increases with frequency and also the characteristic impedance increases slightly with frequency.

Numerical results for the effective dielectric constant and characteristic impedance are given in the following subsection. This algorithm is used to predict higher-order modes in microstrip lines. Results obtained using the present algorithm are in very good agreement with results obtained in [23] for single microstrip lines.

C.4 Results for Effective Dielectric Constant and Characteristic Impedance Using Spectral Domain Method.

The flow chart of the numerical algorithm is given in Table C.1. Results for the effective dielectric constant for a 0.5mm wide microstrip line on $\varepsilon_r = 9.9$, 0.64mm thick are shown in Figure C.3. Results in curve 1 in this figure are obtained using the present algorithm. Curve 2 represents results given in [23]. Results given in curve (3) are obtained using equations given in [43]. One may note that results using the present spectral domain algorithm are the same as spectral domain results given in [23]. Characteristic impedance results for this microstrip line are shown in Figure C.4. Results in curve 1 are obtained by using the present algorithm. One may note that the characteristic impedance decreases slightly as a function of frequency between 2 GHz to 6 GHz. For frequencies higher than 6 GHz the characteristic impedance increases as a function of frequency. Results for curve 2 are given in [23].
Table C.1: Spectral domain numerical algorithm.
Figure C.3: Effective dielectric constant for a microstrip line on ε_r = 9.9, h = 0.635mm, t = 5μm, w = 0.5mm.

The differences in Z_0 between results in curves 1 and 2, spectral domain results, are 0.1Ω at 2 GHz and 1.2Ω at 14 GHz. The differences between results in curve 3, results obtained using equations given in [43], and results in curve 2 are 0.6Ω at 2 GHz and 2.2Ω at 14 GHz. Effective dielectric constant results for a 0.785mm wide microstrip line on ε_r = 2.2 substrate 10 mil thick are shown in Figure C.5. Results given in curve 1 of this figure are obtained using the present algorithm. Results shown in curve 2 are obtained by using equations given in [43]. The differences in the effective dielectric constant results between curves 1 and 2 is less than 0.02. The effective dielectric constant increases with frequency. Results of the characteristic impedance for this case are shown in
Figure C.4: Comparison of computed characteristic impedance for a microstrip line on $\varepsilon_r = 9.9$, $h = 0.635\,mm$, $t = 5\,\mu m$, $w = 0.5\,mm$.

Figure C.6. Results shown in curve 1 are obtained using the present algorithm. Results shown in curve 2 are obtained using equations given in [43]. One may note that $Z_0$ increases slightly with frequency. The differences in $Z_0$ between these two methods are around $0.4\Omega$.

C.5 Higher-Order Modes in Microstrip Lines

A quasistatic approach is not applicable to approximation of the dispersion properties of higher-order modes in microstrip lines. In [28] the cut-off frequency of higher-order modes in microstrip lines was computed using a planar waveguide model and compared with measurements. In [23] the spectral domain
Figure C.5: Comparison of effective dielectric constant results for a 0.785\,mm wide microstrip line on $\varepsilon_r = 2.2$ substrate, 10 mil thick.

The method is applied to calculate the effective dielectric constant of higher-order modes. The present algorithm is also applied to calculate the cut-off frequency and effective dielectric constant of higher-order modes in a 9.17\,mm wide microstrip line on $\varepsilon_r = 9.7$ substrate and 0.64\,mm thick. The present spectral domain algorithm results (curve 1) are compared to results given in [28] (curve 2) and to results given in [23] (curve 3). There is very good agreement between these three sets of results as shown in Figure C.7.

Results of the effective dielectric constant of a microstrip line on $\varepsilon_r = 9.7$ substrate and 0.64\,mm thick with the same width as before for the first and second higher-order modes are shown in Figure C.8. The present spectral domain
Figure C.6: Comparison of characteristic impedance results of a 0.785mm wide microstrip line on \( \varepsilon_r = 2.2 \) substrate, 10 mil thick.

results for the first higher-order mode (curve 1) and for the second higher-order mode (curve 3) are compared to results given in [23] for the first higher-order mode (curve 2) and for the second higher-order mode (curve 4). One may note that there is very good agreement between the present results and results given in [23]. These results verify the present algorithm for calculation of the effective dielectric constant and characteristic impedance of microstrip lines.

Microstrip characteristics used by spectral domain approach (discussed in this appendix) are used in Chapters 7, 8 and 9.
Figure D.6: Even and odd mode characteristic impedance for a coupler on $\varepsilon_r = 2.2$ substrate, 10 mil thick.
by using the present algorithm is 1.760 and the odd mode effective dielectric constant obtained by using Bryant and Weiss algorithm [41] is 1.708. For the same case the odd mode characteristic impedance using the present algorithm is 36.65Ω and the odd mode characteristic impedance obtained using Bryant and Weiss algorithm [41] is 35.19Ω. Results for even and odd mode characteristic impedances for the same coupled lines as in Figure D.5 with spacing between the strips 0.0254mm and 0.8mm, for frequency range from 2 GHz to 18 GHz, are shown in Figure D.6. One may note that $Z_{oe}$ and $Z_{oo}$ values increase slightly with frequency.

![Graph](image)

Figure D.5: Even and odd mode effective dielectric constant for a coupler on $\varepsilon_r = 2.2$ substrate, 10 mil thick.
effective dielectric constant and characteristic impedance increase more rapidly than the odd mode effective dielectric constant and characteristic impedance.

![Graph](image)

Figure D.4: Comparison of $Z_0e$ and $Z_{00}$ for a coupler on $\varepsilon_r = 9.9$ substrate, $h = 0.035mm$ thick.

Effective dielectric constant results for a coupled microstrip line (on $\varepsilon_r = 2.2$, 10 mil thick, with 0.785mm strip width) with spacing between the strips ranging from 0.0254mm to 0.8mm and for frequencies ranging from 2 GHz to 18 GHz are shown in Figure D.5. Comparison of the present results with results obtained by using Bryant and Weiss algorithm [41] at 4 GHz shows a fairly good agreement. As an example, for a coupled microstrip line (on $\varepsilon_r = 2.2$, 10 mil thick substrate, with 0.785mm strip width) with spacing between the strips of 0.0254mm the odd mode effective dielectric constant at 4 GHz obtained
Figure D.3: Even and odd mode effective dielectric constant for a coupled line section on $\varepsilon = 9.9$ substrate, 0.635 mm thick.

One may note that curves 3 and 4 coincide and cannot be distinguished in the figure. Comparison of the even and odd mode characteristic impedances for this case is shown in Figure D.4. $Z_{0e}$ results obtained using the present algorithm are shown as curve 1. $Z_{0e}$ results given in [23] are shown as curve 2. Again there is a very good agreement between these two sets of results. $Z_{0o}$ results obtained by using the present algorithm are shown as curve 3. $Z_{0e}$ results given in [23] are shown as curve 4. Curves 3 and 4 coincide. Results shown in Figures D.3 and D.4 verify the present algorithm. It may be concluded that the even and odd mode characteristic impedance and the even and odd mode effective dielectric constant increase as a function of frequency. The even mode
as

\[
C_{0e} = \frac{1}{\vartheta_{pe} Z_{0e}} \quad (D.3)
\]

\[
C_{00} = \frac{1}{\vartheta_{p0} Z_{00}} \quad (D.4)
\]

where \( \vartheta_{pe} \), \( \vartheta_{p0} \), \( Z_{0e} \), \( Z_{00} \), \( \varepsilon_{re} \), and \( \varepsilon_{ro} \) are incorporated in the spectral domain analysis, the dispersion effects are also included in the planar- lumped model for coupled microstrip line discontinuities by using spectral domain results for \( Z_{0e} \), \( Z_{00} \) and \( \varepsilon_{re} \), \( \varepsilon_{ro} \) in the planar-lumped model analysis described in Chapter 8.

In section D.2 numerical results obtained using the present algorithm are compared to numerical results given in [23].

### D.2 Numerical Results for Coupled Microstrip Lines

The spectral domain approach for coupled microstrip lines described in Section D.1 is used to evaluate coupled line parameters such as even and odd mode effective dielectric constants, \( \varepsilon_{re} \) and \( \varepsilon_{ro} \), and even and odd mode characteristic impedances \( Z_{0e} \), \( Z_{00} \). The present results are compared to results given in [23]. Results for the even and odd mode effective dielectric constants for a coupled microstrip line (on \( \varepsilon_r = 9.9 \), \( 0.64\text{mm} \) thick, the strip width is \( 0.6\text{mm} \) and the spacing between the strips is \( 0.2\text{mm} \) are shown in Figure D.3. Results for the odd mode effective dielectric constant obtained by using the present algorithm are shown as curve 1 of this figure. Results for the odd mode effective dielectric constant given in [23] are shown as curve 2. One may note that there is very good agreement between these results. Results for the even mode effective dielectric constant are shown as curves 3 and 4, where results in curve 3 are obtained by using the present algorithm and results in curve 4 are given in [23].
Figure D.2: Current distribution for coupled microstrip lines.

transforms of (D.1) to obtain the new transforms.

The characteristic impedance $Z_0$ is obtained as

$$Z_0 = \frac{P_{avg}}{I^2} \quad (D.2)$$

since the total effective averaged power surrounding the coupled microstrip lines is now due to two lines. Beyond this change, no major modifications are needed in order to use the algorithm described in Appendix C. For wider microstrip lines and coupled microstrip lines, more than one basis function is needed to represent the current distribution. A set of basis functions for even and odd modes is given in [23].

For given parameters of coupled lines such as spacing, dielectric constant, substrate height, line width and frequency, the output is the even and odd mode effective dielectric constant, and even and odd mode characteristic impedances $Z_{0e}$ and $Z_{0o}$. The even and odd mode capacitance may be calculated
\[ J_z(x) = \frac{1}{\sqrt{\left(\frac{W}{2}\right)^2 - x^2}} \]

\[ J_x(x) = x \sqrt{\left(\frac{W}{2}\right)^2 - x^2} \]

Figure D.1: Current basis functions for the microstrip line.
APPENDIX D

SPECTRAL DOMAIN ANALYSIS FOR EVALUATION OF COUPLED MICROSTRIP LINE PARAMETERS

D.1 Approach Used

The spectral domain algorithm, described in Appendix C, for single microstrip lines, has been extended to analyze coupled microstrip lines. The essential modification for the coupled lines configuration is in the basis functions for the assumed current distribution. Thus, it is necessary only to modify the algorithm described in Appendix C such that the mathematical description of the basis functions corresponds to the physical configuration and current distributions of the coupled lines.

The assumed current distribution for narrow single microstrip lines may be written as

\[ J_x(x) = \frac{1}{\sqrt{\left(\frac{W}{2}\right)^2 - x^2}} \quad (D.1.a) \]

and

\[ J_\varphi(x) = x\sqrt{\left(\frac{W}{2}\right)^2 - x^2} \quad (D.1.b) \]

where \( W \) is the line width, as shown in Figure D.1.

Figure D.2 shows the two assumed current distributions. \( J_x \) for the even and odd modes, respectively. The change in the current component \( J_X \) is similar. In the Fourier domain, one simply applies the shifting theorem to the
Figure C.8: Comparison of effective dielectric constant results for a 9.15mm wide microstrip line on $\varepsilon_r = 9.7$ substrate, 0.635mm thick, for the first and second higher-order mode.
Figure C.7: Comparison of effective dielectric constant for first-order mode with Jansen and Kompa results.
APPENDIX E

DERIVATION OF CAPACITANCE AND INDUCTANCE MATRICES FOR THE PLANAR-LUMPED MODEL FOR COUPLED MICROSTRIPS

Detailed derivation of the capacitance $[C]$ matrix and the inductance $[L]$ matrix for the planar-lumped model described in Chapter 8 are given in this appendix.

E.1 Derivation of capacitance $[C]$ matrix

The lumped network representing the electric field coupling between the two strips is derived from the capacitance matrix of the coupled microstrip line. The capacitance matrix for coupled microstrip lines is obtained from electromagnetic analysis of coupled microstrip lines. Even and odd mode capacitance $C_e$ and $C_0$ respectively) are related to the $[C]$ matrix as follows:

$$[C] = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} = \begin{bmatrix} \frac{1}{2}(C_e + C_0) & \frac{1}{2}(C_e - C_0) \\ \frac{1}{2}(C_e - C_0) & \frac{1}{2}(C_e + C_0) \end{bmatrix}$$  \hspace{1cm} (E.1)

A $\pi$-network representation of the $[C]$ matrix for a coupled line section of length $\Delta t$ is shown in Figure 8.2(a). The even mode capacitance $C_e$ in the $\pi$ network in Figure 8.2(a) is obtained by placing an open circuit at the plane of symmetry of the $\pi$-network. We obtain that $C_e$ is equal to $C_{sh}$. The odd mode capacitance is obtained by placing a short circuit at the plane of symmetry of the $\pi$-network. So the odd mode capacitance is written as

$$C_0 = C_{sh} + 2C_e$$ \hspace{1cm} (E.2)

$$C_e = C_{sh}$$
The relation between the currents and voltages in the π-network of Figure 8.2(a) may be written as

\[
\begin{bmatrix}
i_1 \\
i_2
\end{bmatrix} = j\omega \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}
\] (E.3)

\(C_{11}\) may be obtained by placing a short circuit at port 2, \(C_{11} = \frac{i_1}{V_1}\big|_{V_2=0}\), which gives

\[
i_1 = j\omega(C_{sh} + C_{se})V_1
\] (E.4)

so

\[
C_{11} = C_{sh} + C_{se}
\] (E.5)

\(C_{12}\) is obtained by placing a short circuit at port 1.

\[
C_{12} = \frac{i_1}{V_2}\big|_{V_1=0}
\] (E.6)

\[
i_1 = -j\omega C_{se} V_2
\]

so

\[
C_{12} = C_{21} = -C_{se}
\] (E.7)

Combining equations (E.5) and (E.7) we obtain that

\[
C_e = C_{sh} = C_{11} + C_{12}
\] (E.8)

\[
C_0 = C_{sh} + 2C_{se} = C_{11} - C_{12}
\]

In the planar-lumped model \(C_{sh}\) is partially accounted for by the planar segment (with \([Z] = Z_s\)). The remaining part of \(C_{sh}\) is modeled by the element \(C_f\) of the lumped network representing the coupling gap. A section of this network is shown in Figure 8.2(b). \(C_f\) is found to be

\[
C_f = \left(C_e - \frac{\varepsilon_0 \varepsilon_{re} W_e}{h}\right) \Delta \ell
\] (E.9)

where \(\Delta \ell\) is the length of the line represented by port \(i\). The matrix \([C]\) represents capacitances per unit length of the line.
Kirchoff's laws relating to the network in Figure 8.2(b), may be written as

\[ I_i = j\omega C_f V_i + j\omega (V_i - V_{n+1})C_g \]  \hspace{1cm} (E.10)

\[ I_{n+i} = j\omega C_g (V_{n+i} - V_i) + j\omega C_f V_{n+i} \]  \hspace{1cm} (E.10.b)

From equations (E.10.a) and (E.10.b) the network shown in Figure 8.2.b may be represented by the following Y-matrix:

\[ [Y_c] = j\omega \begin{bmatrix} C_f + C_g & -C_g \\ -C_g & C_f + C_g \end{bmatrix} = j\omega [C_0] \]  \hspace{1cm} (E.11)

The relation between current and voltages in the network section shown in Figure 8.2(b) may be written as

\[ \begin{bmatrix} I_i \\ I_{n+1} \end{bmatrix} = j\omega [Y_c] \begin{bmatrix} V_i \\ V_{n+1} \end{bmatrix} \]  \hspace{1cm} (E.12)

When the planar-lumped model has \( n \) ports on each side, the relation between currents and voltages in the overall [C] network may be written as:

\[ \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \\ I_{n+1} \\ \vdots \\ I_{2n} \end{bmatrix} = j\omega [C_0] \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \\ V_{n+1} \\ \vdots \\ V_{2n} \end{bmatrix} \]  \hspace{1cm} (E.13)

where the complete C-matrix may be written as
E.2 Derivation of Inductance Matrix

The magnetic field coupling between the two strips is modeled by mutual inductance elements in the lumped network. The inductive coupling network for a coupled line section of length $\Delta \ell$ is shown in Figure 8.3(a) and Figure 8.3(b). The total inductance network is shown in Figure 8.3.c. Values of $L_p$ and $M$ are calculated by comparing the $[L]$ matrix of the coupled line with that for the modeling network.

The inductance matrix for the coupled line pair is obtained from the capacitance matrix $[C_0]$ for the case when the dielectric is replaced by air. The relation between $[L]$ and $[C_0]$ may be written as \[44\]

\[
[L] = \mu_0 \varepsilon_0 [C_0]^{-1} = \begin{bmatrix}
L_{11} & L_{12} \\
L_{21} & L_{22}
\end{bmatrix}
\]  

(E.15)

where $\mu_0$ and $\varepsilon_0$ are permeability and permittivity of free space, respectively.
The network representation of the inductance matrix \([L]\) for length \(\Delta \ell\) of a coupled line pair is shown in Figure 8.3(b). A part of the \(\Delta \ell\) \(L_{11}\) is contributed by the inductance of the parallel plate waveguide and the remaining by \(L_p\) as shown in Figures 8.3(a) and 8.3(c).

The capacitance matrix \([C_0]\) for the case when the dielectric is replaced by air may be written as

\[
[C_0] = \begin{bmatrix}
\frac{C_{a}^{\text{air}} + C_{e}^{\text{air}}}{2} & \frac{C_{a}^{\text{air}} - C_{e}^{\text{air}}}{2} \\
\frac{C_{a}^{\text{air}} - C_{e}^{\text{air}}}{2} & \frac{C_{a}^{\text{air}} + C_{e}^{\text{air}}}{2}
\end{bmatrix}
\]  

(E.16)

where

\[
C_{a}^{\text{air}} = \frac{C_e}{\varepsilon_{re}} \\
C_{0}^{\text{air}} = \frac{C_0}{\varepsilon_{r0}}
\]

where \(\varepsilon_{re}\) and \(\varepsilon_{r0}\) are effective dielectric constants for the even and odd modes respectively. Substituting (E.16) in (E.15), the \([L]\) matrix may be written as

\[
\begin{bmatrix}
L_{11} & L_{12} \\
L_{21} & L_{22}
\end{bmatrix} = \mu_0 \varepsilon_0 \begin{bmatrix}
\frac{C_{a}^{\text{air}} + C_{0}^{\text{air}}}{2} & \frac{C_{a}^{\text{air}} - C_{0}^{\text{air}}}{2} \\
\frac{C_{a}^{\text{air}} - C_{0}^{\text{air}}}{2} & \frac{C_{a}^{\text{air}} + C_{0}^{\text{air}}}{2}
\end{bmatrix}^{-1}
\]  

(E.17)

The relation between currents and voltages in the network shown in Figure 8.3(c) may be written as

\[
\begin{bmatrix}
I_1 \\
I_2
\end{bmatrix} = \frac{1}{j \omega} \left( \begin{bmatrix} L_p & M \\ M & L_p \end{bmatrix}^{-1} + \begin{bmatrix} L_{pt} & 0 \\ 0 & L_{pt} \end{bmatrix}^{-1} \right) \begin{bmatrix} \Delta V_1 \\ \Delta V_2 \end{bmatrix}
\]  

(E.18)

Equation (E.18) may be written as

\[
\begin{bmatrix}
I_1 \\
I_2
\end{bmatrix} = \frac{1}{j \omega} [Y_i] \begin{bmatrix} \Delta V_1 \\ \Delta V_2 \end{bmatrix}
\]  

(E.19)

The relation between currents and voltages in the network shown in Figure 8.3(b) may be written as

\[
\begin{bmatrix}
I_1 \\
I_2
\end{bmatrix} = \frac{1}{j \omega} \begin{bmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{bmatrix}^{-1} \begin{bmatrix} \Delta V_1 \\ \Delta V_2 \end{bmatrix} = \frac{1}{j \omega} [L]^{-1} \begin{bmatrix} \Delta V_1 \\ \Delta V_2 \end{bmatrix}
\]  

(E.20)
Equating terms in equations (E.18) and (E.20), \( L_p \) and \( M \) are expressed in terms of \( L_{pt} \) and \( [L] \) as follows:

\[
L_p = \frac{L_{12} - (L_{11}^2 - L_{12}^2)/L_{pt}}{1 - 2L_{11}/L_{pt} + (L_{11}^2 - L_{12}^2)/L_{pt}^2} \Delta \ell \quad \text{(E.21.a)}
\]

and

\[
M = \frac{L_{12} \Delta \ell}{1 - 2L_{11}/L_{pt} + (L_{11}^2 - L_{12}^2)/L_{pt}^2} \Delta \ell \quad \text{(E.21.b)}
\]

In (E.21.a) and (E.21.b) it is assumed that \( L_{11} = L_{22} \) and that \( L_{12} = L_{21} \) (symmetric lines). Formulation could, of course, be extended to asymmetric lines also. The inductive part of the total lumped network is shown in Figure 8.4. The \((2n \times 2n)\) admittance matrix of this network is obtained by using Kirchhoff's laws. Referring to Figure 8.4, the relations between voltage and currents in the network shown in Figure 8.4 may be written as

\[
V_1 - V_2 = j\omega L_p \ I'_2 + j\omega M \ I_{n+2} \quad \text{(E.22.a)}
\]

\[
V_{n+2} - V_{n+1} = j\omega M \ I'_2 + j\omega L_p \ I_{n+2} \quad \text{(E.22.b)}
\]

\[
V_3 - V_2 = j\omega L_p \ I'_3 + j\omega M \ I_{n+3} \quad \text{(E.23.a)}
\]

\[
V_{n+3} - V_{n+2} = j\omega M \ I'_3 + j\omega L_p \ I_{n+3} \quad \text{(E.23.b)}
\]

By multiplying equation (E.22.a) with \( j\omega L_p \) and equation (E.22.b) with \( j\omega M \) and subtracting the result we can obtain \( I'_2 \).

\[
I'_2 = \frac{j}{\omega(L_p^2 - M^2)} (L_p V_2 - L_p V_1 - M V_{n+2} + M V_{n+1}) \quad \text{(E.24.a)}
\]

By multiplying equation (E.22.1) with \( j\omega M \) and equation (E.22.b) with \( j\omega L_p \) and subtracting the result, \( I'_{n+2} \) is obtained.

\[
I'_{n+2} = \frac{j}{-\omega(M^2 - L_p^2)} (M V_2 - M V_1 - L_p V_{n+2} + L_p V_{n+1}) \quad \text{(E.24.b)}
\]
By employing the same procedure on equations (E.23.a) and (E.23.b) we obtain \( I_3 \) and \( I'_{n+3} \).

\[
I_3' = \frac{j}{\omega(L_p^2 - M^2)} (L_p V_3 - L_p V_2 - MV_{n+3} + MV_{n+2}) \tag{E.25.a}
\]

\[
I'_{n+3} = \frac{j}{\omega(M^2 - L_p^2)} (MV_3 - MV_2 - L_p V_{n+3} + L_p V_{n+2}) \tag{E.25.b}
\]

Using the same procedure at all the ports yields expressions for the currents in terms of the voltages in the adjacent ports. We get

\[
V_1 - V_2 = j\omega L_p l_1 + j\omega M I_{n+1} \tag{E.26.a}
\]

\[
V_{n+1} - V_{n+2} = j\omega M I_1 + j\omega M I_{n+1} \tag{E.26.b}
\]

\[
I_1 = \frac{j}{-\omega(L_p^2 - M^2)} (L_p V_1 - L_p V_2 - MV_{n+1} + MV_{n+2}) \tag{E.27.a}
\]

\[
I_{n+1} = \frac{j}{-\omega(M^2 - L_p^2)} (MV_1 - MV_2 - L_p V_{n+1} + L_p V_{n+2}) \tag{E.27.b}
\]

\[
V_2 - V_3 = j\omega L_p l''_2 + j\omega M I''_{n+2} \tag{E.28.a}
\]

\[
V_{n+2} - V_{n+3} = j\omega M I''_2 + j\omega L_p I''_{n+2} \tag{E.28.b}
\]

\[
I''_2 = \frac{j}{-\omega(L_p^2 - M^2)} (L_p V_2 - L_p V_3 - MV_{n+2} + MV_{n+3}) \tag{E.29.a}
\]

\[
I''_{n+2} = \frac{j\omega}{-\omega(L_p^2 - M^2)} (MV_1 - MV_3 - L_p V_{n+2} + L_p V_{n+3}) \tag{E.29.b}
\]

\[
I_2 = I''_2 + I''_2 = \frac{j}{-\omega(L_p^2 - M^2)} (2L_p V_2 - L_p V_3 - 2MV_{n+2} + MV_{n+3} - L_p V_1 + MV_{n+1}) \tag{E.30.a}
\]

\[
I_{n+2} = I'_{n+2} + I''_{n+2} = \frac{j}{-\omega(L_p^2 - M^2)} (MV_1 - 2MV_2 + MV_3 + 2L_p V_{n+2} - L_p V_{n+1} - L_p V_{n+3}) \tag{E.30.b}
\]

From the circuit in Figure 8.3 we can conclude that

\[
I_2 = I_3 = I_4 = \ldots = I_{n-1}
\]

\[
I_{n+2} = I_{n+3} = \ldots = I_{2n-1}
\]

\[
I_1 = I_n \text{ and } I_{n+1} = I_{2n}
\]
So the overall matrix \([L_G]\) may be written as

\[
\begin{bmatrix}
1 & 2 & 3 & \ldots & n-1 & n & n+1 & n+2 & n+3 & \ldots & 2n-1 & 2n \\
\vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\
L_p & L_p & 0 & -L_p & 2L_p & -L_p & \cdots & -L_p & 2L_p & -L_p & 0 & L_p \\
-2L_p & -L_p & 0 & L_p & 2L_p & -L_p & \cdots & L_p & 2L_p & -L_p & 0 & L_p \\
-2L_p & -L_p & 0 & L_p & 2L_p & -L_p & \cdots & L_p & 2L_p & -L_p & 0 & L_p \\
\vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\
-2M & M & 0 & -2M & M & 0 & \cdots & -2M & M & 0 & -2M & M \\
0 & 0 & M & -2M & M & -2M & \cdots & 0 & 0 & M & -2M & M \\
0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

(E.32)

In the inductive network shown in Figure 8.4, half inductance sections at the terminal ports \((1, n, n+1, 2n)\) are not included. A more accurate modeling should incorporate these sections as shown in Figure 8.5. Thus, the lumped network representing the gap becomes a \((2n + 4)\) port network and the \([L_G]\) matrix needs to be modified.

The number of ports in the gap region needed for convergence of coupling results was decided by iterative computations. Optimum width of the ports interconnecting the rectangular segments to the gap network is found to be \(\lambda/88\) (when half inductance sections at terminating ports are not included). This number can be reduced to about \(\lambda/52\) when half inductance sections at four ends are included and the derivation of the modified \(L\) matrix is given in this Appendix.
For the modified $L$-network shown in Figure 8.5 a similar procedure is used to derive the $L$-matrix, only the equations that relate the voltages to the current at the ports $1$, $2$, $n + 1$, $n + 3$, $n + 4$, $2n + 3$ and $2n + 4$ are modified as follows. Referring to Figure 8.5 the relations between the voltages and currents may be written as

\[ V_1 - V_2 = \frac{j\omega L_p}{2} I'_3 + \frac{j\omega M}{2} I'_{n+4} \]  
(E.33.a)

\[ V_{n+3} - V_{n+4} = \frac{j\omega M}{2} I'_3 + \frac{j\omega M}{2} I'_{n+4} \]  
(E.33.b)

By multiplying equation (E.33.a) with $j\omega L_p$ and equation (E.33.b) with $j\omega M$ and subtracting, we can obtain $I'_3$.

\[ I'_3 = \frac{j}{\omega (L^2_p - M^2)} (2L_p V_2 - 2L_p V_1 + 2M V_{n+3} - 2M V_{n+4}) \]  
(E.34.a)

In a similar manner $I'_{n+4}$ may be written as

\[ I'_{n+4} = \frac{j}{-\omega (M^2 - L^2_p)} (2M V_2 - 2M V_1 - 2L_p V_{n+4} + 2L_p V_{n+3}) \]  
(E.34.b)

\[ V_3 - V_2 = j\omega L_p I'_3 + j\omega M I'_{n+5} \]  
(E.35.a)

\[ V_{n+5} - V_{n+4} = j\omega L_p I'_3 + j\omega L_p I'_{n+5} \]  
(E.35.b)

$I'_3$ and $I'_{n+5}$ may be written as

\[ I'_3 = \frac{-j}{\omega (L^2_p - M^2)} (L_p V_3 - L_p V_2 - M V_{n+5} - M V_{n+4}) \]  
(E.36.a)

\[ I'_{n+5} = \frac{j}{-\omega (M^2 - L^2_p)} (M V_3 - M V_2 - L_p V_{n+5} + L_p V_{n+4}) \]  
(E.36.b)

A similar procedure is used to find the relation between the currents and voltages in all the ports.

\[ V_1 - V_2 = \frac{j\omega L_p}{2} I_1 + \frac{j\omega M}{2} I_{n+3} \]  
(E.37.a)

\[ V_{n+3} - V_{n+4} = \frac{j\omega M}{2} I_1 + \frac{j\omega M}{2} I_{n+3} \]  
(E.37.b)
Using equations (E.30.a) and (E.30.b), \( I_1 \) and \( I_{n+3} \) may be written as

\[
I_1 = \frac{j}{-\omega (L_p^2 - M^2)} (2L_p V_1 - 2L_p V_2 - 2M V_{n+3} + 2M V_{n+4}) \tag{E.38.a}
\]

\[
I_{n+3} = \frac{j}{-\omega (M^2 - L_p^2)} (2M V_1 - 2M V_2 - 2L_p V_{n+3} + 2L_p V_{n+4}) \tag{E.38.b}
\]

\[
V_2 - V_3 = j \omega L_p I_2'' + j \omega M I_{n+4}'' \tag{E.39.a}
\]

\[
V_{n+4} - V_{n+5} = j \omega M I_{n+4}'' + j \omega L_p I_{n+4}'' \tag{E.39.b}
\]

\( I_2'' \) and \( I_{n+4}'' \) may be written as

\[
I_2'' = \frac{j}{-\omega (L_p^2 - M^2)} (L_p V_2 - L_p V_3 = M V_{n+4} + M V_{n+5}) \tag{E.40.a}
\]

\[
I_{n+4}'' = \frac{j}{-\omega (M^2 - L_p^2)} (M V_2 - M V_3 - L_p V_{n+4} + L_p V_{n+5}) \tag{E.40.b}
\]

\[
I_2 = I_2'' + I_2'' = \frac{j}{-\omega (L_p^2 - M^2)} (3L_p V_2 - 2L_p V_1 - L_p V_3 - 3M V_{n+4} + 2M V_{n+3} + M V_{n+5}) \tag{E.41}
\]

\[
I_{n+4} = I_{n+4}'' + I_{n+4}'' = \frac{j}{-\omega (L_p^2 - M^2)} (3M V_2 - M V_3 - 2M V_1 - 3L_p V_{n+4} + 2L_p V_{n+3} + L_p V_{n+5}) \tag{E.42}
\]

From symmetry, the modified \([L_G]\) matrix may be written as shown in equation E.43.
Modified $(L)$-matrix for modeling $H$-field coupling.
APPENDIX F

CONVERGENCE OF S-PARAMETERS FOR THE PLANAR-LUMPED
MODEL OF A COUPLED LINE SECTION AS A FUNCTION OF
THE NUMBER OF PORTS IN THE GAP REGION

In the planar-lumped model for coupled lines, described in Chapter 8, the two planar waveguide segments are connected together through a multi-port lumped network. The number of ports needed to ensure matching of the fields under the strips to those in the gap region is obtained by checking the convergence of S-parameter results as a function of the number of ports in the gap region. Convergence of S-parameter results as a function of the number of ports in the gap region of a quarter-wave section of an ideal coupler is investigated. The configuration considered is a pair of microstrip lines (each 50Ω characteristic impedance in isolation) on 10 mil thick substrate with \( \varepsilon_r = 2.2 \) and their lengths equal to \( \frac{\lambda}{4} \) at 10 GHz, the spacing between the strips is 0.127 mm. S-parameter results, obtained by using the \([L]\) network given in Table 8.2, for different numbers of ports in the gap region, are listed in Table F.1.

One may note that the differences in \( S_{11}, S_{12} \) and \( S_{14} \) when 11, 21 and 23 ports are taken, are less than 0.3 dB, 0.01 dB and 0.04 dB respectively. The difference in \( S_{13} \) when 11 ports are considered and when 23 ports are taken is 3.88 dB. One may conclude that in order to obtain convergence in \( S_{11}, S_{12} \) and \( S_{14} \) we need only 11 ports. In order to obtain more accurate results and convergence in \( S_{13} \) we need around 21 to 23 ports. For loose couplings, the
Table F.1: $S$-parameter results in dB for an ideal coupler with $(s/h = 0.5)$ for different numbers of ports in the gap region.

<table>
<thead>
<tr>
<th></th>
<th>11 Ports</th>
<th>21 Ports</th>
<th>23 Ports</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_{11}$</td>
<td>-30.370</td>
<td>-30.187</td>
<td>-30.05</td>
</tr>
<tr>
<td>$S_{12}$</td>
<td>-15.78</td>
<td>-15.77</td>
<td>-15.78</td>
</tr>
<tr>
<td>$S_{13}$</td>
<td>-30.55</td>
<td>-27.9</td>
<td>-26.77</td>
</tr>
<tr>
<td>$S_{14}$</td>
<td>-0.152</td>
<td>-0.156</td>
<td>-0.158</td>
</tr>
</tbody>
</table>

The convergence of $S$-parameters as a function of the number of ports in the gap region is better as shown in Table F.2 for a -19.2 dB coupler and in Table F.3 for a -49.3 dB coupler.

Table F.2: $S$-parameter results in dB for an ideal coupler with $(s/h = 1)$ for different numbers of ports in the gap region.

<table>
<thead>
<tr>
<th></th>
<th>11 Ports</th>
<th>21 Ports</th>
<th>26 Ports</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_{11}$</td>
<td>-39.300</td>
<td>-30.090</td>
<td>-38.98</td>
</tr>
<tr>
<td>$S_{13}$</td>
<td>-30.580</td>
<td>-29.000</td>
<td>-28.400</td>
</tr>
<tr>
<td>$S_{14}$</td>
<td>-8.455 $10^{-2}$</td>
<td>-8.660 $10^{-2}$</td>
<td>8.750 $10^{-2}$</td>
</tr>
</tbody>
</table>

Table F.3: $S$-parameter results in dB for an ideal coupler with $(s/h = 15.75)$ for different numbers of ports in the gap region.

<table>
<thead>
<tr>
<th></th>
<th>11 Ports</th>
<th>17 Ports</th>
<th>27 Ports</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_{11}$</td>
<td>-52.26</td>
<td>-52.25</td>
<td>-52.24</td>
</tr>
<tr>
<td>$S_{12}$</td>
<td>-49.39</td>
<td>-49.36</td>
<td>-49.35</td>
</tr>
<tr>
<td>$S_{13}$</td>
<td>-54.22</td>
<td>-53.69</td>
<td>-53.29</td>
</tr>
<tr>
<td>$S_{14}$</td>
<td>-2.82 $10^{-2}$</td>
<td>-2.82 $10^{-2}$</td>
<td>-2.82 $10^{-2}$</td>
</tr>
</tbody>
</table>
One may note that the difference in $S_{11}$, when 11 ports and when 26 ports are taken, is 0.33 dB. The difference in $S_{12}$ when 11 ports in the gap region are taken and when 26 ports in the gap region are taken, is 0.044 dB. Isolation $S_{13}$ is much more sensitive to the number of ports in the gap region. The difference in isolation $S_{13}$ results when 11 ports are taken and when 26 ports are taken, is 2.16 dB. One may conclude that for this case good convergence is obtained for $S_{11}$, $S_{12}$ and $S_{14}$ with eleven ports. In order to obtain good convergence in $S_{13}$ we need about 26 ports. $S$-parameter results for the same coupler when the spacing between the strips is $4\text{mm}$ are given in Table F.3.

Again, one can see that when 11 ports are taken there is very good convergence in $S_{11}$, $S_{12}$ and $S_{14}$. The difference in $S_{13}$ results, isolation, when 11 ports are taken and when 27 ports are taken in the gap region is 0.93 dB.

Usually in coupler design $S_{11}$, $S_{12}$ and $S_{14}$ are the important parameters and accuracy in isolation of $\pm 1\text{ dB}$ may be acceptable. One may conclude that eleven ports in the gap region may be good enough to obtain accurate results in the design of couplers using the planar-lumped model for coupled microstrip line discontinuities. However, in our computations reported in this thesis, one hundred ports per wavelength are taken to ensure matching of the fields under the strips to those in the gap region.
APPENDIX G

DERIVATION OF SOURCE-FREE AMPLITUDE TERMS
AND OF SOURCE TERMS

The normalized amplitude terms $H_1$ through $E_3$ of equations (10.9) and (10.10) are derived in this appendix. Also, the derivation of the source terms $B_{ie}$ and $B_{ih}$ is presented.

G.1 Derivation of Source-Free Amplitude Terms

The normalized amplitude terms ($H_1$ through $E_3$), are derived in terms of two field amplitudes, $E_{1h}$ and $H_{1e}$ by applying Maxwell's source-free equations:

$$\nabla \times E = -j\omega \mu H$$  \hspace{1cm} (G.1)

First we will derive expressions for amplitude terms in a closed cavity. The source-free $E$ field equation (10.10) is substituted into equation (G.1) and the curl operation is performed to yield

$$H_1 = \frac{(a_2 E_3 - a_3 E_2)}{j \omega \mu}$$
$$H_2 = \frac{(a_3 E_1 - a_1 E_3)}{j \omega \mu}$$
$$H_3 = \frac{(a_1 E_2 - a_2 E_1)}{j \omega \mu}$$  \hspace{1cm} (G.2)

where $H_1$, $H_2$, $H_3$ are the amplitude terms of the magnetic field and where $a_1$, $a_2$ and $a_3$ are given in equation (10.3.b).

Using Maxwell's source-free divergence equation $\nabla \cdot E = 0$ in equation (10.10) results in

$$a_1 E_1 + a_2 E_2 + a_3 E_3 = 0$$  \hspace{1cm} (G.3)
The TE mode terms may be evaluated by setting $E_{3h} = 0$. Combining equations (G.2) and (G.3) and expressing the amplitudes in terms of only one field variable, $E_{1h}$, yields the following equations:

$$
E_{2h} = -\frac{\alpha_1}{\alpha_2} E_{1h}
$$

$$
H_{1h} = \frac{1}{j\omega \mu} \left( \frac{\alpha_1 \alpha_3}{\alpha_2} \right) E_{1h}
$$

$$
H_{2h} = \frac{1}{j\omega \mu} \alpha_3 E_{1h}
$$

$$
H_{3h} = -\frac{1}{j\omega \mu} \left( \frac{\alpha_1^2 + \alpha_2^2}{\alpha_2} \right) E_{1h}
$$

\hspace{2cm} (G.4)

The subscripts $h$ indicate TE or H mode. Applying $\nabla \times H = j\omega \varepsilon E$ to equation (10.9) results in

$$
E_1 = \frac{(-\alpha_2 H_3 + \alpha_3 H_2)}{j\omega \varepsilon}
$$

$$
E_2 = \frac{(-\alpha_3 H_1 + \alpha_1 H_3)}{j\omega \varepsilon}
$$

$$
E_3 = \frac{(-\alpha_1 H_2 + \alpha_2 H_1)}{j\omega \varepsilon}
$$

\hspace{2cm} (G.5)

Now, applying $\nabla \cdot B = 0$ to equation (10.9) we get

$$
\alpha_1 H_1 + \alpha_2 H_2 + \alpha_3 H_3 = 0
$$

\hspace{2cm} (G.6)

The TM modes are evaluated by setting $H_{3e} = 0$ and combining equations (G.5) and (G.6) to obtain equation (G.7).

$$
H_{2e} = -\frac{\alpha_1}{\alpha_2} H_{1e}
$$

$$
E_{1e} = \frac{1}{j\omega \varepsilon} \left( \frac{-\alpha_1 \alpha_3}{\alpha_2} \right) H_{1e}
$$

$$
E_{2e} = \frac{1}{j\omega \varepsilon} (-\alpha_3) H_{1e}
$$

$$
E_{3e} = \frac{1}{j\omega \varepsilon} \left( \frac{\alpha_1^2 + \alpha_2^2}{\alpha_2} \right) H_{1e}
$$

\hspace{2cm} (G.7)

The subscript $e$ indicates TM or E-mode.

The source-free amplitudes ($H_1$ through $E_3$) are the superposition of the TE and TM modes (i.e. $E_1 = E_{1h} + E_{1e}$). The amplitude terms may be written
\[ H_1 = H_{1e} - \frac{j}{\omega \mu} \left( \frac{\alpha_1 \alpha_3}{\alpha_2} \right) E_{1h} \]

\[ H_2 = -\frac{\alpha_1}{\alpha_2} H_{1e} - \frac{j}{\omega \mu} \alpha_3 E_{1h} \]  \hspace{1cm} (G.8)

\[ H_3 = \frac{j}{\omega \mu} \left( \alpha_1^2 / \alpha_2 + \alpha_3^2 / \alpha_2 \right) E_{1h} \]

\[ E_1 = E_{1h} + \frac{j}{\omega \epsilon} \left( \frac{\alpha_1 \alpha_3}{\alpha_2} \right) H_{1e} \]

\[ E_2 = -\frac{\alpha_1}{\alpha_2} E_{1h} + \frac{j}{\omega \epsilon} \alpha_3 H_{1e} \]  \hspace{1cm} (G.9)

\[ E_3 = -\frac{j}{\omega \epsilon} \left( \alpha_1^2 / \alpha_2 + \alpha_3^2 / \alpha_2 \right) H_{1e} \]

G.2 Fields in an Open (No Side Walls) Cavity

The open cavity has electrical conductors at \( Z = 0, Z = h \) and magnetic walls at the side walls. The boundary conditions for the magnetic walls are

\[ E_{\text{norm}} = 0 \]

\[ H_{\text{tan}} = 0 \]  \hspace{1cm} (G.10)

When these boundary conditions are applied to the electromagnetic fields, the following equations are obtained:

\[ H_i = \bar{a}_x \ H_x + \bar{a}_y \ H_y + a_z \ H_z \]  \hspace{1cm} (G.11)

where

\[ H_x = H_1 \cos(\alpha_1 x) \sin(\alpha_2 y) \cos(\alpha_3 z) \]

\[ H_y = H_2 \sin(\alpha_1 x) \cos(\alpha_2 y) \cos(\alpha_3 z) \]

\[ H_z = H_3 \sin(\alpha_1 x) \sin(\alpha_2 y) \sin(\alpha_3 z) \]

\[ E_i = \bar{a}_x \ E_x + \bar{a}_y \ E_y + a_z \ E_z \]  \hspace{1cm} (G.12)

where

\[ E_x = E_1 \sin(\alpha_1 x) \cos(\alpha_2 y) \sin(\alpha_3 z) \]

\[ E_y = E_2 \cos(\alpha_1 x) \sin(\alpha_2 y) \sin(\alpha_3 z) \]

\[ E_z = E_3 \cos(\alpha_1 x) \cos(\alpha_2 y) \cos(\alpha_3 z) \]
The derivation of the source-free amplitude terms in an open cavity follows the same steps as in the closed cavity using equations (G.11) and (G.12). The source-free amplitudes \( H_1 \) through \( H_3 \) for the open cavity may be written as

\[
H_1 = H_{1e} + \frac{j}{\omega \mu} \left( \frac{\alpha_1 \alpha_3}{\alpha_2} \right) E_{1h}
\]

\[
H_2 = -\frac{\alpha_1}{\alpha_2} H_{1e} + \frac{j}{\omega \mu} \alpha_3 E_{1h}
\]

\[
H_3 = \frac{j}{\omega \varepsilon} \left( \frac{\alpha_1^2 + \alpha_3^2}{\alpha_2} \right) E_{1h}
\]

\[
E_1 = E_{1h} + \frac{j}{\omega \varepsilon} \left( \frac{\alpha_1 \alpha_3}{\alpha_2} \right) H_{1e}
\]

\[
E_2 = -\frac{\alpha_1}{\alpha_2} + \frac{j}{\omega \varepsilon} \alpha_3 H_{1e}
\]

\[
E_3 = \frac{j}{\omega \varepsilon} \left( \frac{\alpha_1^2 + \alpha_3^2}{\alpha_2} \right) H_{1e}
\]

(G.13)

G.3 Normalizing Factors \( H_{1e}, E_{1h} \)

Using the power normalization equations [52],

\[
\int_V \mu H_i \cdot H_i^* dV = 1
\]

\[
\int_V \varepsilon E_i \cdot E_i^* dV = 1
\]

we perform the scalar product in equation (G.14) to obtain

\[
\int_V \mu (|H_x|^2 + |H_y|^2 + |H_z|^2) dV = 1
\]

\[
\int_V \varepsilon (|E_x|^2 + |E_y|^2 + |E_z|^2) dV = 1
\]

(G.15)

Using equations (10.9) and (10.10) and integrating over the cavity volume gives

\[
\mu_0 \int_0^a \int_0^b \int_0^h (H_1 H_i^* \sin^2 \alpha_1 x \cos^2 \alpha_2 y \cos^2 \alpha_3 z + H_2 H_i^* \cos^2 \alpha_1 x \sin^2 \alpha_2 y \cos^2 \alpha_3 z + H_3 H_i^* \cos^2 \alpha_1 x \cos^2 \alpha_2 y \sin^2 \alpha_3 z) \right) dz dy dx = 1
\]

(G.16)

These integrals can be evaluated using

\[
\int \sin^2 \alpha x \ dx = -\frac{\cos \alpha x \sin \alpha x}{2a} + \frac{x}{2}
\]

\[
\int \cos^2 \alpha x \ dx = \frac{\cos \alpha x \sin \alpha x}{2a} + \frac{x}{2}
\]

(G.17)
where $\alpha_1 = \frac{m\pi}{\mu}$, $\alpha_2 = \frac{n\pi}{\nu}$ and $\alpha_3 = \frac{p\pi}{\epsilon}$. The solution of equation (G.16) for $m \neq 0$, $n \neq 0$ and $p \neq 0$ may be written as
\[
\mu H_1^2 \frac{\alpha_1^2}{8} + \mu H_2^2 \frac{\alpha_2^2}{8} + \mu H_3^2 \frac{\alpha_3^2}{8} = 1 \tag{G.18}
\]
The power normalization equation may be written as
\[
H_1^2 + H_2^2 + H_3^2 = \frac{B_{mnp}^2}{\mu \alpha \beta h} \tag{G.19}
\]
\[
E_1^2 + E_2^2 + E_3^2 = \frac{B_{mnp}^2}{\epsilon \alpha \beta h}
\]
where $B_{mnp}$ may be written as
\[
B_{mnp} = \begin{cases} \sqrt{8} & m \neq 0, \ n \neq 0, \ p \neq 0 \\ \frac{2}{\text{one of } m, \ n, \ p = 0} \end{cases} \tag{G.20}
\]
If $m = n = p = 0$ the fields given in equations (10.9) and (10.10) are equal to zero. If two of indices $m, \ n, \ p$ equal zero, there is no solution of Maxwell’s equations. Substituting equations (G.8) and (G.9) into equation (G.19) gives
\[
\frac{B_{mnp}^2}{\mu \alpha \beta h} = H_1^2 \left( 1 + \left( \alpha_1 \frac{\alpha_2^2}{\alpha_2^2} \right) \right) + \frac{1}{(\omega \mu)^2} \left[ \left( \frac{\alpha_1 \alpha_3}{\alpha_2^2} \right)^2 + \alpha_2^2 + \right. \\
\left. \left( \frac{\alpha_1^2 + \alpha_3^2}{\alpha_2^2} \right)^2 \right] E_1^2 \tag{G.21}
\]
\[
\frac{B_{mnp}^2}{\epsilon \alpha \beta h} = E_1^2 \left( 1 + \left( \frac{\alpha_1}{\alpha_2^2} \right) \right) + \frac{1}{(\omega \epsilon)^2} \left[ \left( \frac{\alpha_1 \alpha_3}{\alpha_2^2} \right)^2 + \alpha_2^2 + \right. \\
\left. \left( \frac{\alpha_1^2 + \alpha_3^2}{\alpha_2^2} \right)^2 \right] H_1^2 \tag{G.22}
\]
Combining (G.21) and (G.22) yields
\[
H_1^2 = Q \left[ \left( \frac{\nu - T}{\nu \epsilon} \right)^2 \frac{G_2}{S^2 - \left( \frac{T^2}{\mu \epsilon \nu} \right)} \right]^{1/2} \tag{G.23}
\]
\[
E_1^2 = \left[ \frac{1}{S} \left( \frac{\nu - T}{\epsilon \nu} \right)^2 \frac{G_2}{H_1^2} \right]^{1/2} \tag{G.24}
\]
where
\[
Q = \frac{B_{mnp}}{\sqrt{\alpha \beta h}}
\]
\[
S = \left( 1 + \left( \frac{\alpha_1}{\alpha_2^2} \right)^2 \right)
\]
\[
R = \left( \frac{\alpha_1 \alpha_3}{\alpha_2^2} \right)^2 \alpha_2^2 + \left( \frac{\alpha_1^2}{\alpha_2^2} \right)^2 + 2 \alpha_1^2 + \alpha_2^2
\]
\[
T = \frac{R}{\omega^2}
\]
When cavity modes with no variation in the \( y \) direction, \( n = 0 \) modes, are considered, special care needs to be taken in calculating the amplitude terms of the source-free fields. When \( n \) equals zero, \( \alpha_2 \) is also equal to zero and this creates undesired singularities in equations (G.8) through (G.11). This problem may be avoided by setting \( \alpha_2 \) equal to zero in equations (10.9), (10.10), (G.11), (G.12). For closed cavity these equations may be written as

\[
H_x = H_1 \sin(\alpha_1 x) \cos(\alpha_3 z) \\
H_y = 0 \\
H_z = H_3 \cos(\alpha_1 x) \sin(\alpha_3 z) \\
E_x = 0 \\
E_y = E_2 \sin(\alpha_1 x) \sin(\alpha_3 z) \\
E_z = 0
\]  

(G.25)

Now applying Maxwell’s equations to (G.25) and (G.26) results in

\[
H_1 = \frac{j}{\omega \mu} \alpha_3 E_{2h} \\
H_2 = H_{2e} \\
H_3 = \frac{-j}{\omega \mu} \alpha_1 E_{2h} \\
E_1 = \frac{j}{\omega \epsilon} \alpha_3 H_{2e} \\
E_2 = E_{2h} \\
E_3 = \frac{j}{\omega \epsilon} \alpha_1 H_{2e}
\]  

(G.28)

For open cavity the amplitude terms for \( m = 0 \), may be written as

\[
H_x = 0 \\
H_y = H_2 \sin(\alpha_1 x) \cos(\alpha_3 z) \\
H_z = 0 \\
E_x = E_1 \sin(\alpha_1 x) \sin(\alpha_3 z) \\
E_y = 0 \\
E_z = E_3 \cos(\alpha_1 x) \cos(\alpha_3 z)
\]  

(G.29)

(G.30)
\[ H_1 = \frac{-j}{\omega \mu} \alpha_3 E_{2h} \]

\[ H_2 = H_{2e} \quad (G.31) \]

\[ H_3 = \frac{-j}{\omega \varepsilon} \alpha_3 H_{2h} \]

\[ E_1 = \frac{j}{\omega \varepsilon} \alpha_3 H_{2e} \]

\[ E_2 = E_{2h} \quad (G.32) \]

\[ E_3 = -\frac{j}{\omega \varepsilon} \alpha_1 H_{2e} \]

The normalization factors \( E_{2h} \) and \( H_{2e} \) can be calculated by substituting equations (G.27) and (G.28) into equations (G.17) as follows:

\[
\left( \frac{1}{\omega \mu} \right)^2 \alpha_3^2 E_{2h}^2 + H_{2e}^2 + \left( \frac{1}{\omega \varepsilon} \right)^2 \alpha_1^2 E_{2h}^2 = \frac{B_{mn}^2}{\mu abh} \quad (G.33) \\
\left( \frac{1}{\omega \varepsilon} \right)^2 \alpha_3^2 H_{2e}^2 + E_{2e}^2 + \left( \frac{1}{\omega \varepsilon} \right)^2 \alpha_1^2 H_{2e}^2 = \frac{B_{mn}^2}{\varepsilon abh}
\]

The solution of equation (G.33) results in the following expressions for \( E_{2h} \) and \( H_{2e} \):

\[ E_{2h} = \left[ \frac{1}{\varepsilon} (Q^2 - X \mu H_{2e}^2) \right]^{1/2} \]

\[ H_{2e} = \left[ \frac{Q^2 (1 - X)}{\mu (1 - X^2)} \right]^{1/2} \quad (G.34) \]

where

\[ X = \frac{\alpha_3^2 + \alpha_1^2}{\omega^2 \mu \varepsilon} \]

\[ Q = \frac{B_{mn}^2}{\varepsilon abh} \]

\[ B_{mn} = 2 \quad \text{for } n = 0 \quad \text{as in equation (G.20)} \]

### G.4 Derivation of Source Terms

The source terms \( B_{ie} \) and \( B_{ih} \) of equations (10.6) and (10.7) include the effect of the magnetic current element in the cavity:

\[ B_{ie} = \frac{j \omega_i}{\omega^2 - \omega_i^2} \int_V M \cdot H_i^* \, dV \]

\[ B_{ih} = \frac{j \omega}{\omega^2 - \omega_i^2} \int_V M \cdot H_i^* \, dV \quad (G.35) \]
Performing the scalar product in (G.35), equation (G.35) may be written as

\[
B_{ie} = \frac{j \omega_i}{\omega_0^2 - \omega_i^2} (B_x + B_y) \\
B_{ih} = \frac{j \omega_i}{\omega_0^2 - \omega_i^2} (B_x + B_y)
\]

\hspace{1cm} (G.36)

where

\[
B_x = \int_V M_x H_z^* \, dV \\
B_y = \int_V M_y H_y^* \, dV
\]

\hspace{1cm} (G.37)

\[\omega = 2\pi f\]

where \( f \) is the operating frequency and \( \omega_i \) is the mode angular frequency.

\[\omega_i^2 = \frac{1}{\mu \varepsilon} (a_1^2 + a_2^2 + a_3^2)\]

\hspace{1cm} (G.38)

The magnetic current element \( M \) may have \( x \) and \( y \) components centered at \( x = a_0, \ y = b_0 \) and \( z = h_0 \). The \( x \) component of the magnetic current, \( M_x \), is directed parallel to the \( x \) axis. It has length \( d\ell_x \) and infinitesimal width. The \( y \) component of the magnetic current, \( M_y \), is directed parallel to the \( y \) axis. It has length \( d\ell_y \) and infinitesimal width. The amplitude of the magnetic current element is \( M_0 \).

Using the source-free magnetic field for the closed cavity equation (10.9) and performing the integration in equation (G.37) over the area of the magnetic current element yields

\[
B_x = \frac{2 \, H_i^* M_0 \cos(a_3 h_0)}{a_1} \sin(a_1 a_0) \sin(a_1 \frac{d\ell_x}{2}) \cos(a_2 b_0)
\]

\hspace{1cm} (G.39)

\[
B_y = \frac{2 \, H_i^* M_0 \cos(a_3 h_0)}{a_2} \sin(a_2 b_0) \sin(a_2 \frac{d\ell_y}{2}) \cos(a_1 a_0)
\]

\hspace{1cm} (G.40)

When \( a_1 \) is equal to zero, \( B_x \) equal to zero and \( B_y \) may be written as

\[
B_y = \frac{2 \, H_i^* M_0 \cos(a_3 h_0)}{a_2} \sin(a_2 b_0) \sin(a_2 \frac{d\ell_y}{2})
\]

\hspace{1cm} (G.41)

When \( a_2 \) is equal to zero, \( B_y \) is equal to zero and \( B_x \) may be written as

\[
B_x = \frac{2 \, H_i^* M_0 \cos(a_3 h_0)}{a_1} \sin(a_1 a_0) \sin(a_1 \frac{d\ell_x}{2})
\]

\hspace{1cm} (G.42)
$B_x$ and $B_y$ for open cavity using the source-free magnetic field for the open cavity, equation (G.11) and performing the integration in equation (G.37) over the area of the magnetic current elements yields

$$B_x = \frac{2 H_r^1 M_0 \cos(\alpha_3 h_0)}{\alpha_1} \sin(\alpha_2 b_0) \cos(\alpha_1 a_0) \sin(\alpha_1) \frac{d\ell_x}{2}$$  \hspace{1cm} (G.43)

$$B_y = \frac{2 H_r^2 M_0}{\alpha_2} \cos(\alpha_3 h_0) \sin(\alpha_1 a_0) \cos(\alpha_2 b_0) \sin(\alpha_2) \frac{d\ell_y}{2}$$  \hspace{1cm} (G.44)

When $\alpha_1$ is equal to zero, $B_y$ is equal to zero and $B_x$ may be written as

$$B_x = 0$$ \hspace{1cm} (G.45)

$$B_x = H_r^1 M_0 d\ell_x (\cos(\alpha_3 h_0) \sin(\alpha_2 b_0))$$

When $\alpha_2$ is equal to zero, $B_x$ is equal to zero and $B_y$ may be written as

$$B_x = 0$$ \hspace{1cm} (G.46)

$$B_y = H_r^2 M_0 d\ell_y (\cos(\alpha_3 h_0) \sin(\alpha_1 a_0))$$