**Homework Project 4 – FFT Beam Propagation**

**Numerical Methods in Photonics**

**ECEN 6006**

**Project**

The goal of this project is to model propagation in weakly refracting but continuously inhomogeneous media. It builds strongly on the Fourier free-space propagation assignment.

**Problem 1**
An easy one to get started. Implement a quadratic-profile gradient index lens and verify its properties. Saleh and Teich section 1.3 is a good reference. In specific, use an index profile $n^2(x) = n_0^2(1 + a^2 y^2)$ (S&T equation 1.3-7) and show that a rod of length $d$ has a focal length of $\sim 1/(n_0 a \sin (ad))$ for $d < \pi/(2a)$. The only tricky part of this is figuring out how to extract “focal length” from a diffraction calculation. The formal definition from ray tracing is $F = y_0 / -u_k$ where $y_0$ is the height of a axial (parallel to the axis) ray entering the lens and $u_k$ is the angle of that ray at the exit of the lens. Think about how you can establish this with the information in a BPM simulation (it’s there). Be careful to avoid over-filling the lens and causing significant spherical aberration.

I’ve got an extra copy of Saleh if you need one.

**Problem 2**
Implement a Kerr nonlinearity, as shown in the notes. This means that at each step, compute the electric field in real space, then calculate the instantaneous nonlinearity that gives you the index, then refract through this index. Show stable soliton propagation using the sech form given in the notes. Hint: be careful with units in this equation!

**Problem 3**
Implement and test an absorbing boundary condition. Experiment with different parameters to see how thickness and max absorption affect reflectivity.

**Problem 4**
Volume holography. Fill a portion of your space with a weak, thick transmission hologram (a sinusoidal index perturbation). Think in advance about what incident and diffracted angles and hologram index, period and angle you want. If it makes sense, duplicating your k-space calculation of this same problem would be great – you can compare results in the weakly diffracting regime. Demonstrate Bragg matching by propagating through this hologram and generating a single diffracted order, then changing the angle enough to just Bragg mismatch which should then not diffract. To see the diffracted angles coming off of the grating, it’s typical in the lab to use a Fourier transform lens – you have a very easy way to use this method here.
Saleh or any similar optics text will describe this situation if you’re not familiar with it, or you can use the k-space formalism we developed in class.

Extras if you’re feeling energetic: Try propagation in a curved space or TM propagation or a sharp bend or 2+1 dimensions or any of the other goodies on the notes (or not in the notes!).