**Homework Project 5 – Waveguide eigenmode propagation and Coupled modes**

**Numerical Methods in Photonics**

ECEN 6006

**Project part 1 (slabs)**

The goal of this project is to exercise and compare the three methods we have for strongly inhomogeneous media in the SVEA limit (that is, only forward-going waves).

In the notes, the problem of two coupled waveguides is solved 3 different ways. In this assignment, we will replicate these results.

The problem specification is two slab waveguides of 8 microns width (a=4 µm) separated by 10 microns (center to center, so 2 micron gap) with cladding index 1.5 and core index 1.51, wavelength 1 um.

**Problem 1 – Beam propagation method**

Solve the problem using your BPM code and extract the coupling length – the distance in which 100% of the energy couples to the second guide. This will work best if you launch a mode into one guide that is close to the fundamental. Use one of the methods of the notes to find this initial field.

**Problem 2 – Modal propagation**

I have written and executed a multilayer slab waveguide solver (as discussed in class) for you. The mode profiles are stored in four files posted with this homework.

In Matlab, execute:

```matlab
E0 = load('Emode0.mat');
E0 = E0.Expression7;  % This is necessary due to writing the files in mathematica
E1 = load('Emode1.mat');
E1 = E1.Expression6;
E2 = load('Emode2.mat');
E2 = E2.Expression5;
E3 = load('Emode3.mat');
E3 = E3.Expression2;
```

Or in Mathematica, execute:

```mathematica
E0 = Import["Emode0.MAT"];
E1 = Import["Emode1.MAT"]; 
E2 = Import["Emode2.MAT"]; 
E3 = Import["Emode3.MAT"]; 
```
One column of the array is the electric field amplitude; one is the coordinate in microns (it runs from -20 to +20 microns in .1 micron steps).

The effective indices of the four modes are:

- 1.509188732476186
- 1.5091135620856033
- 1.5068223867458135
- 1.506493327464166

Since we have just the guided modes, our results will automatically and immediately exclude radiation modes. Given this restriction, make a plot (like in the notes) that shows the spatial distribution of guided fields when one guide is excited with the same input used in the first problem. The notes used just the lowest two modes – do this and then include all four and see how the results compare to the BPM. What is the coupling length in this case (this is the exact solution)?

**Problem 3 – Coupled modes**

In this case we need the mode of a single guide (the unperturbed problem). We’ll use just the lowest order mode.

In Matlab execute:

```matlab
Esingle0 = load('EmodeSingle0.mat');
Esingle0 = E3.Expression8;
```

Or in Mathematica

```mathematica
Esingle0 = Import["EmodeSingle0.MAT"];
```

The effective index in this case is 1.5091485950625152. Note that the effective indices of the two lowest order modes of the coupled system are split around this value. Think about effective index as a weighted average of the index with mode power and this should make sense. This is the fundamental math by which a single bound mode (e.g an electron wave in a solid-state material) becomes a “band” of modes when many identical potential wells are periodically spaced. Use the coupled mode method to predict how one guide will couple into another and make a plot of the efields vs space as in the notes. Predict the coupling length.

**Project part 2 (ridges)**

Download Tom Murphy’s mode solver from the matlab file exchange website. Read and understand the example code coupler_even_odd.m. Run this to see the even and odd fundamental TE modes of this ridge guide. With this solution…
Problem 1 – BPM. Stretch #1: If you have a 2+1D BPM, give this a try, but it’s not required. Use the anti-symmetric solution as an approximation of the fundamental mode of one guide and launch this into two parallel guides. Extract the coupling length.

Problem 2 – Modal propagation. Using the results of coupler_even_odd, make a plot of the coupler in action via an image which is a slice through the x-z plane at the center of the ridge. What is the coupling length?

Problem 3 – Coupled modes. Stretch #2: Modify the coupler_even_odd code to find the mode of an isolated single ridge. Calculate the overlap integral numerically to find the coupling constant and from this the coupling length.