Problem 1 (15 pts). Four identical light-bulbs are connected to a voltage source, as shown. If bulb one is observed to emit $P_1 = 1$ watt of power in the form of light, how much power is emitted by bulbs 2, 3 and 4? That is, what are $P_2$, $P_3$ and $P_4$?

The radiated power will be less than the total dissipated power, but they must be proportional. Assume some total voltage $v$ is provided by the source and that each bulb has a resistance $R$. Then $P_1 = 1 \, [W] = \frac{v^2}{R}$.

5 pts: Bulb 2 is in parallel with bulb 1 so the voltages are equal, thus $P_2 = P_1 = 1 \, [W]$

10 pts: Bulbs 3 and 4 are in series and, with equal resistances, must divide the voltage $v$ into two equal parts. Thus $P_{3,4} = \frac{(v/2)^2}{R} = \frac{v^2}{R} / 4 = \frac{P_1}{4} = 0.25 \, [W]$
Problem 2 (15 pts). A battery is connected to a load of resistance $R$ at time $= 0$. The current is observed to begin at a value $i_0$ and then to decrease as shown below. How much total energy and charge were delivered to the load? Show your work including any fundamental relations you are using (don’t just write down the answer).

Current is (for $t < T$)

$$i(t) = i_0 \quad (t < T/2), \quad = 2i_0 (1-t/T) \quad (T/2 < t < T)$$

Power delivered to load is

$$p(t) = Ri^2(t)$$

Voltage across load is

$$v(t) = Ri(t)$$

Charge delivered to load is time integral of current

$$q = \int_0^T i(t) dt = \int_0^{T/2} i(t) dt + \int_{T/2}^T i(t) dt = i_0 T/2 + 2i_0 \int_{T/2}^T \left(1 - \frac{t}{T}\right) dt$$

$$= i_0 T/2 + 2i_0 \left[ \frac{t - t^2}{2T} \right]_{T/2}^T = i_0 T/2 + 2i_0 \left( \frac{T - T}{2} - \frac{T - T}{2} + \frac{T}{8} \right) = \frac{3}{4} i_0 T \quad [C]$$

Energy delivered to load is time integral of power

$$E = \int_0^{T/2} p(t) dt + \int_{T/2}^T p(t) dt = Ri_0^2 T/2 + R4i_0^2 \int_{T/2}^T \left(1 - \frac{t}{T}\right)^2 dt = \frac{2}{3} Ri_0^2 T / 2 + \frac{2}{3} Ri_0^2 T / 6 = \frac{2}{3} Ri_0^2 T \quad [J]$$

Note that $3/4$ is expected from area of graph, above, just via simple geometry.

Reasonable: $2/3$ is a bit less than $3/4$ because quadratic curve $T/2$ to $T$ has bit less area under it than triangle.
Problem 3 A (10 pts): Use the equivalence principal to reduce the left circuit to the right one, that is, find $R_{eq}$. Do any required algebra to simplify the result.

\[ R_{eq} = (2R + (4R \parallel 4R \parallel 4R) + 2R) \parallel (5R + 5R) \parallel (5R + 5R) \]
\[ = (4R + R) \parallel 10R \parallel 10R \]
\[ = \left( \frac{1}{5R} + \frac{1}{10R} + \frac{1}{10R} \right)^{-1} \]
\[ = \frac{5}{2} R \]

Unchanged from test.

Problem 3 B (5 pts): CIRCLE ONE: Identical currents are driven into both circuits. The voltage measured across each set of terminals is the

a) The same
b) Different
c) There is not enough information to say

Problem 3 C (5 pts): CIRCLE ONE: Identical voltages are connected across both circuits. The power dissipated in the various resistors has which of the following relationships:

a) Each resistor in the original circuit dissipates less power than the power dissipated in $R_{eq}$.
b) Some of the resistors in the original circuit dissipate less and some more than the power dissipated in $R_{eq}$.
c) Each resistor in the original circuit dissipates more power than the power dissipated in $R_{eq}$.
d) There is insufficient information to determine this relationship.
Problem 4 (30 pts): Using the partially-labeled circuit below, set up the complete node voltage solution in its final form. Do not solve the equations, but write equations that, if given the resistor and voltage source values, would be ready for numerical solution.

Finish labels 5 pts

Put ground at node C so that node B is at source voltage. $v_B = v_s \quad$ 5 pts

$v_C = 0 \quad$ 5 pts

Write KCL at each node.

A : $-i_1 - i_2 = 0 \quad$ 8 pts

D : $-i_4 + i_5 = 0 \quad$ 8 pts

E : $+i_1 + i_3 - i_5 = 0 \quad$ 8 pts

Write Ohm's law for each resistor

$i_1 = G_1 v_1 = G_1 (v_A - v_E) \quad$ 8 pts

$i_2 = G_2 v_2 = G_2 (v_A - v_B) \quad$ 8 pts

$i_3 = G_3 v_3 = G_3 (v_C - v_E) \quad$ 8 pts

$i_4 = G_4 v_4 = G_4 (v_D - v_C) \quad$ 8 pts

$i_5 = G_5 v_5 = G_5 (v_E - v_D) \quad$ 8 pts

Substitute in

A : $-G_1 (v_A - v_E) - G_2 (v_A - v_B) = 0 \quad$ 4 pts

D : $-G_4 (v_D - v_C) + G_5 (v_E - v_D) = 0 \quad$ 4 pts

E : $+G_1 (v_A - v_E) + G_3 (v_C - v_E) - G_5 (v_E - v_D) = 0 \quad$ 4 pts
Problem 5 (20 pts): Draw the iv relationship for the pictured components. Label as much of the graph as you can with relevant quantities.

Switch. Draw and label iv curves for both open and closed cases.

Current of voltage source is undetermined. Adding another current in parallel doesn't change that.

This is just like having a resistor in parallel with the voltage source.